

Chapter 6

Hash-Based Indexing

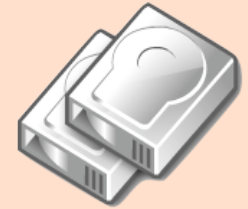
Efficient Support for Equality Search

Architecture and Implementation of Database Systems

Winter 2010/11

Hash-Based Indexing

Torsten Grust



Hash-Based Indexing

Static Hashing

Hash Functions

Extendible Hashing

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Procedures

Linear Hashing

Insertion (Split, Rehashing)

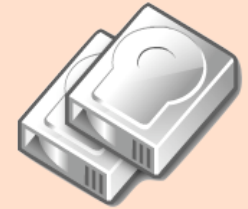
Running Example

Procedures

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Universität Tübingen



Hash-Based Indexing



- We now turn to a different family of index structures: **hash indexes**.
- Hash indexes are “unbeatable” when it comes to support for **equality selections**:

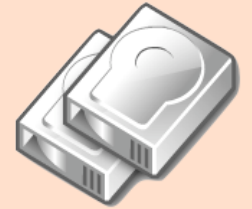
Equality selection

```
1 SELECT *  
2 FROM R  
3 WHERE A = k
```

- Further, other query operations internally generate a flood of equality tests (e.g., nested-loop join).
(Non-)presence of hash index support can make a real difference in such scenarios.

Hashing vs. B⁺-trees

- Hash indexes provide **no support for range queries**, however (hash indexes are also known as **scatter storage**).
- In a B⁺-tree-world, to locate a record with key k means to **compare** k with other keys k' organized in a (tree-shaped) search data structure.
- Hash indexes **use the bits of k itself** (independent of all other stored records) to find the location of the associated record.
- We will now briefly look into **static hashing** to illustrate the basics.
 - Static hashing does *not* handle updates well (much like ISAM).
 - Later, we introduce **extendible hashing** and **linear hashing** which refine the hashing principle and adapt well to record insertions and deletions.



Static Hashing

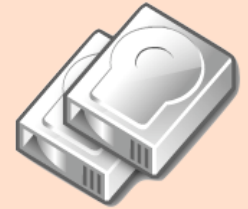
- To build a **static hash index** on attribute A:

Build static hash index on column A

- 1 Allocate a fixed area of N (successive) disk pages, the so-called **primary buckets**.
- 2 In each bucket, install a pointer to a chain of **overflow pages** (initially set the pointer to **null**).
- 3 Define a **hash function** h with *range* $[0, \dots, N - 1]$. The *domain* of h is the type of A, e.g..

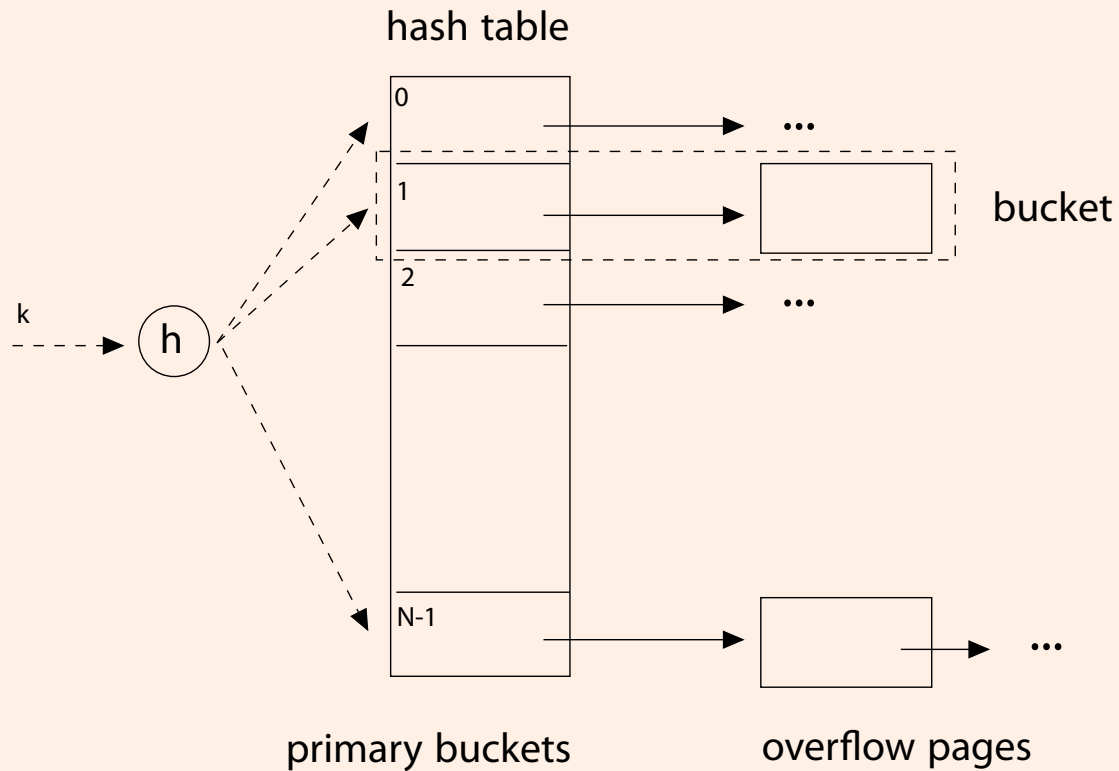
$$h : \text{INTEGER} \rightarrow [0, \dots, N - 1]$$

if A is of SQL type INTEGER.

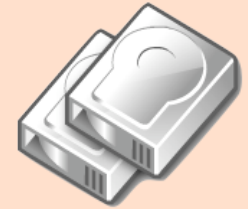


Static Hashing

Static hash table



- A primary bucket and its associated chain of overflow pages is referred to as a **bucket** (□□□ above).
- Each bucket contains **index entries** k^* (implemented using any of the variants **A**, **B**, **C**, see slide 0.0).



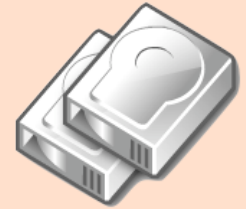
Static Hashing

- To perform $hsearch(k)$ (or $hinsert(k)/hdelete(k)$) for a record with key $A = k$:

Static hashing scheme

- 1 **Apply hash function h** to the key value, *i.e.*, compute $h(k)$.
- 2 **Access the primary bucket page** with number $h(k)$.
- 3 Search (insert/delete) subject record on this page or, if required, **access the overflow chain** of bucket $h(k)$.

- If the hashing scheme works well and overflow chain access is avoidable,
 - $hsearch(k)$ requires a **single I/O operation**,
 - $hinsert(k)/hdelete(k)$ require **two I/O operations**.



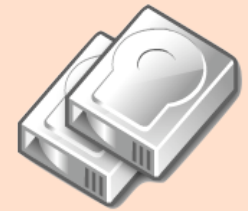
Static Hashing: Collisions and Overflow Chains

- At least for static hashing, **overflow chain management** is important.
- Generally, we do **not** want hash function h to avoid **collisions**, *i.e.*,

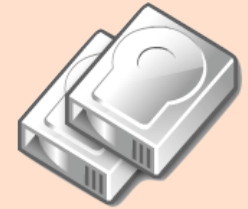
$$h(k) = h(k') \quad \text{even if} \quad k \neq k'$$

(otherwise we would need as many primary bucket pages as different key values in the data file).

- At the same time, we want h to **scatter** the key attribute domain **evenly** across $[0, \dots, N - 1]$ to avoid the development of long overflow chains for few buckets. This makes the hash tables' I/O behavior non-uniform and unpredictable for a query optimizer.
- Such “good” hash functions are hard to discover, unfortunately.



The Birthday Paradox (Need for Overflow Chain Management)



Example (The birthday paradox)

Consider the people in a group as the **domain** and use their birthday as **hash function** h ($h : Person \rightarrow [0, \dots, 364]$).

*If the group has 23 or more members, chances are $> 50\%$ that two people share the same birthday (**collision**).*

Check: Compute the probability that n people *all* have different birthdays:

```
1 Function: different_birthday ( $n$ )
2 if  $n = 1$  then
3   return 1;
4 else
5   return  $\underbrace{\text{different\_birthday}(n - 1)}_{\text{probability that } n-1 \text{ persons have different birthdays}} \times \underbrace{\frac{365 - (n - 1)}{365}}_{\text{probability that } n\text{th person has birthday different from first } n - 1 \text{ persons}} ;$ 
```


Hash Functions

- It is impossible to generate truly random hash values from the non-random key values found in actual table. Can we define hash functions that scatter even better than a random function?

Hash function

- 1 **By division.** Simply define

$$h(k) = k \bmod N .$$

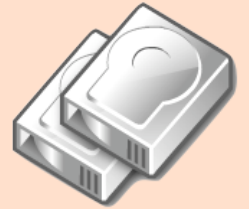
This guarantees the range of $h(k)$ to be $[0, \dots, N - 1]$.

Note: Choosing $N = 2^d$ for some d effectively considers the least d bits of k only. **Prime numbers** work best for N .

- 2 **By multiplication.** Extract the fractional part of $Z \cdot k$ (for a specific Z^1) and multiply by arbitrary hash table size N :

$$h(k) = \lfloor N \cdot (Z \cdot k - \lfloor Z \cdot k \rfloor) \rfloor$$

¹The (inverse) **golden ratio** $Z = 2/(\sqrt{5}+1) \approx 0.6180339887$ is a good choice. See D.E.Knuth, "Sorting and Searching."

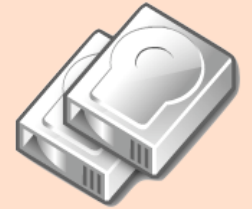


Static Hashing and Dynamic Files

- For a static hashing scheme:
 - If the underlying **data file grows**, the development of overflow chains spoils the otherwise predictable behavior hash I/O behavior (1–2 I/O operations).
 - If the underlying **data file shrinks**, a significant fraction of primary hash buckets may be (almost) empty—a waste of page space.
- As in the ISAM case, however, static hashing has advantages when it comes to concurrent access.
- We may periodically **rehash** the data file to restore the ideal situation (20 % free space, no overflow chains).
 - ⇒ Expensive and the index cannot be used while rehashing is in progress.

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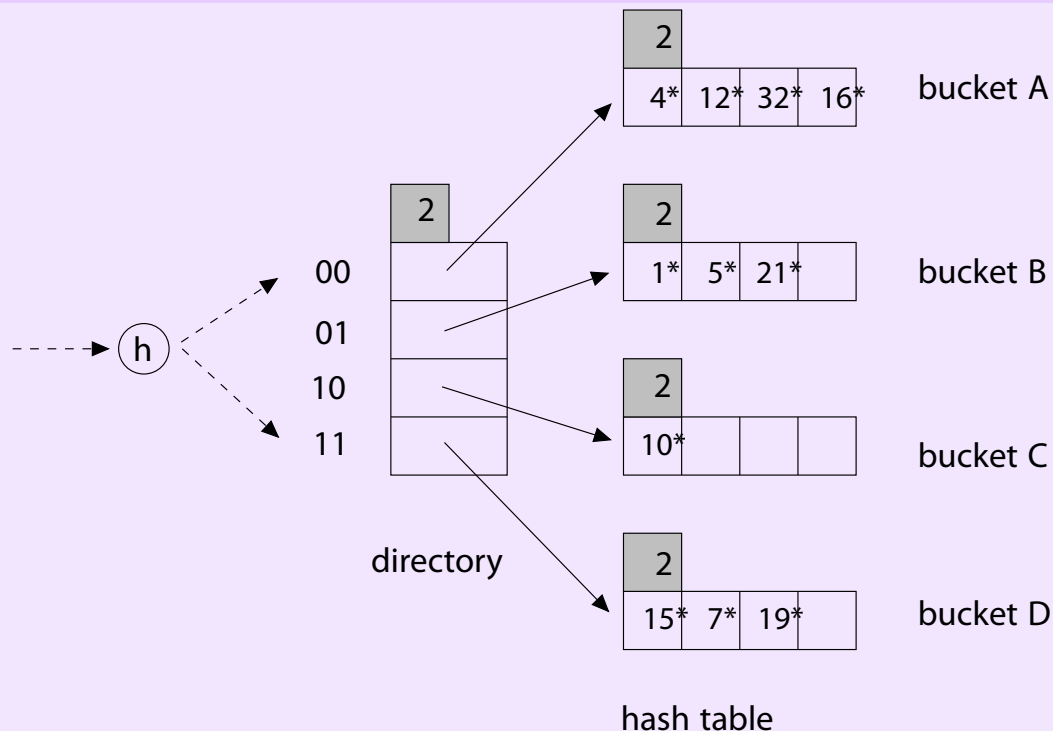
Running Example

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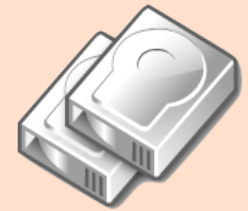
Extendible Hashing

- **Extendible Hashing** can adapt to growing (or shrinking) data files.
- To keep track of the actual primary buckets that are part of the current hash table, we hash via an **in-memory bucket directory**:

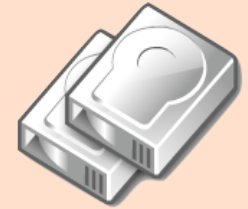
Example (Extendible hash table setup; ignore the **2** fields for now²)



²**Note:** This figure depicts the entries as $h(k)*$, not $k*$.



Extendible Hashing: Search



Search for a record with key k

- 1 Apply h , *i.e.*, compute $h(k)$.
- 2 Consider the last **2** bits of $h(k)$ and follow the corresponding directory pointer to find the bucket.

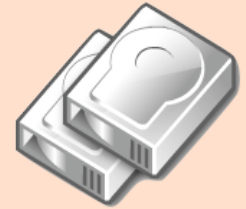
Example (Search for a record)

To find a record with key k such that $h(k) = 5 = 101_2$, follow the second directory pointer ($101_2 \wedge 11_2 = 01_2$) to bucket B, then use entry 5^* to access the wanted record.

Extendible Hashing: Global and Local Depth

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Global and local depth annotations

- **Global depth** (n at hash directory):
Use the last n bits of $h(k)$ to lookup a bucket pointer in the directory (the directory size is 2^n).

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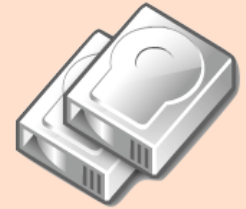
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Extendible Hashing: Global and Local Depth

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Global and local depth annotations

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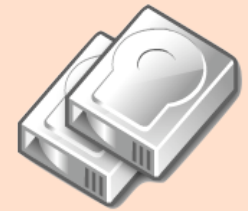
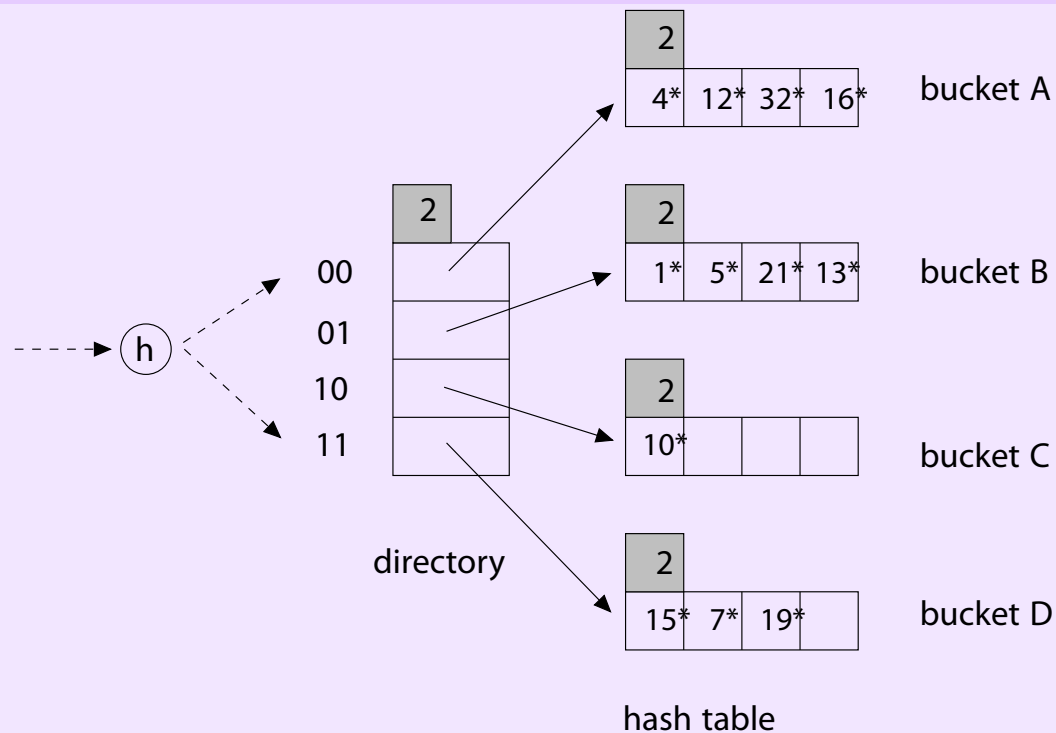
- **Global depth** (n at hash directory):
Use the last n bits of $h(k)$ to lookup a bucket pointer in the directory (the directory size is 2^n).
- **Local depth** (d at individual buckets):
The hash values $h(k)$ of all entries in this bucket agree on their last d bits.

Extendible Hashing: Insert

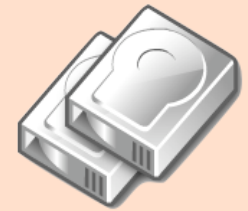
Insert record with key k

- 1 Apply h , i.e., compute $h(k)$.
- 2 Use the last n bits of $h(k)$ to lookup the bucket pointer in the directory.
- 3 If the *primary bucket* still has capacity, store $h(k)^*$ in it. (Otherwise ...?)

Example (Insert record with $h(k) = 13 = 1101_2$)



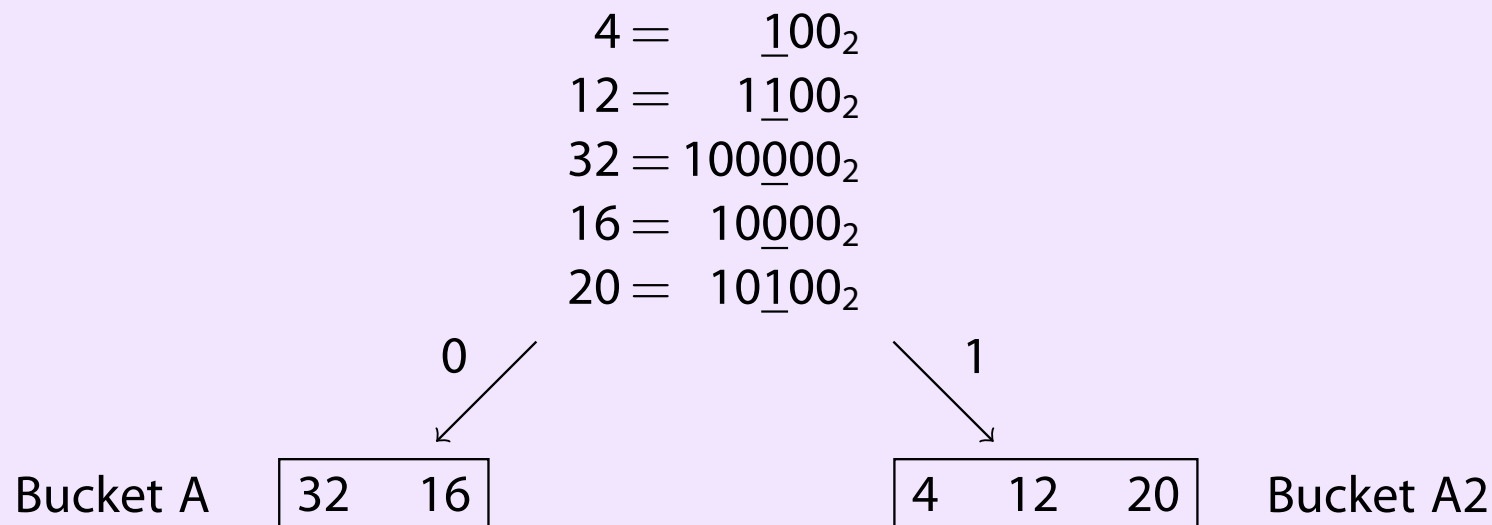
Extendible Hashing: Insert, Bucket Split



Example (Insert record with $h(k) = 20 = 101\underline{00}_2$)

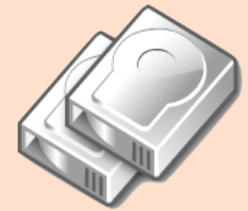
Insertion of a record with $h(k) = 20 = 101\underline{00}_2$ leads to **overflow in primary bucket A**. Initiate a **bucket split** for A.

- 1 **Split** bucket A (creating a new bucket A2) and use bit position $\boxed{d} + 1$ to redistribute the entries:



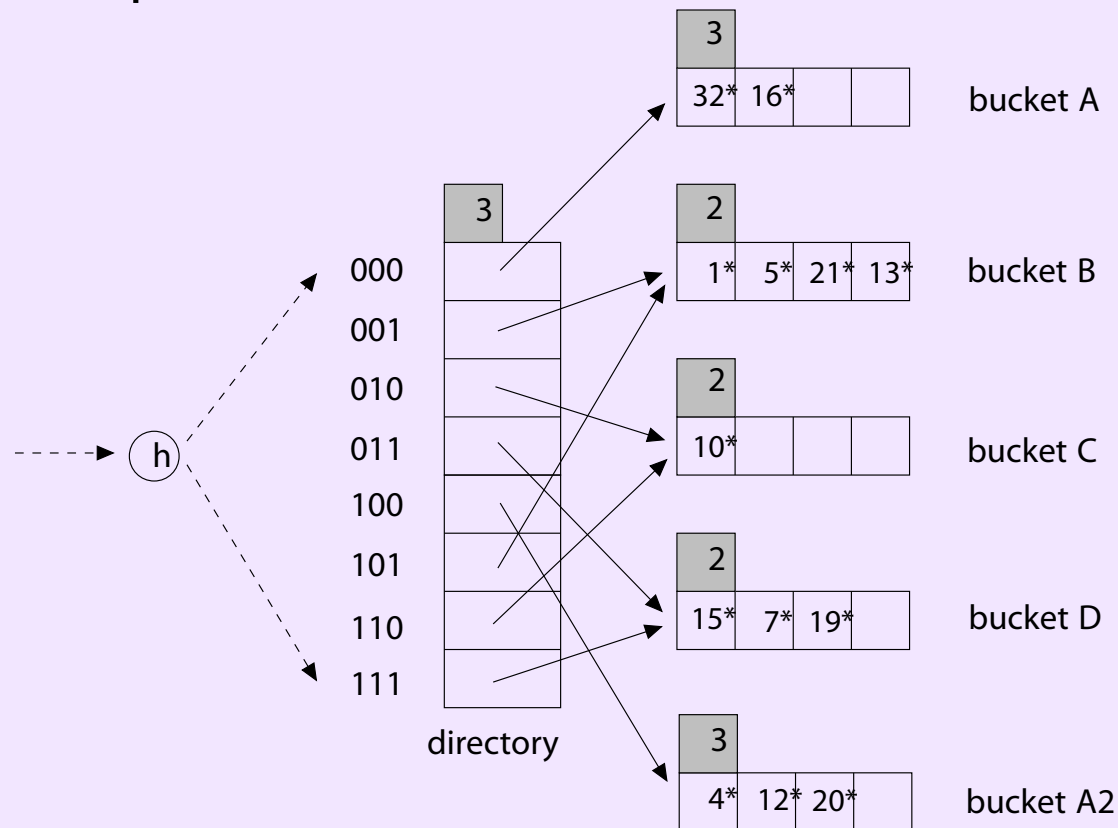
Note: We now need $\boxed{3}$ bits to discriminate between the old bucket A and the new split bucket A2.

Extendible Hashing: Insert, Directory Doubling



Example (Insert record with $h(k) = 20 = 101\underline{00}_2$)

- 2 In the present case, we need to **double the directory** by simply copying its original pages (we now use $\boxed{2} + 1 = \boxed{3}$ bits to lookup a bucket pointer).
- 3 Let bucket pointer for $\underline{1}00_2$ point to A2 (the directory pointer for $\underline{0}00_2$ still points to bucket A):



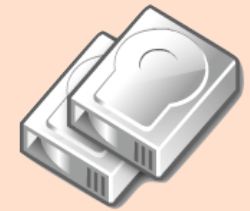
Extendible Hashing: Insert

If we split a bucket with local depth $d < n$ (global depth), directory doubling is *not* necessary:



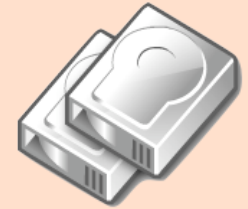
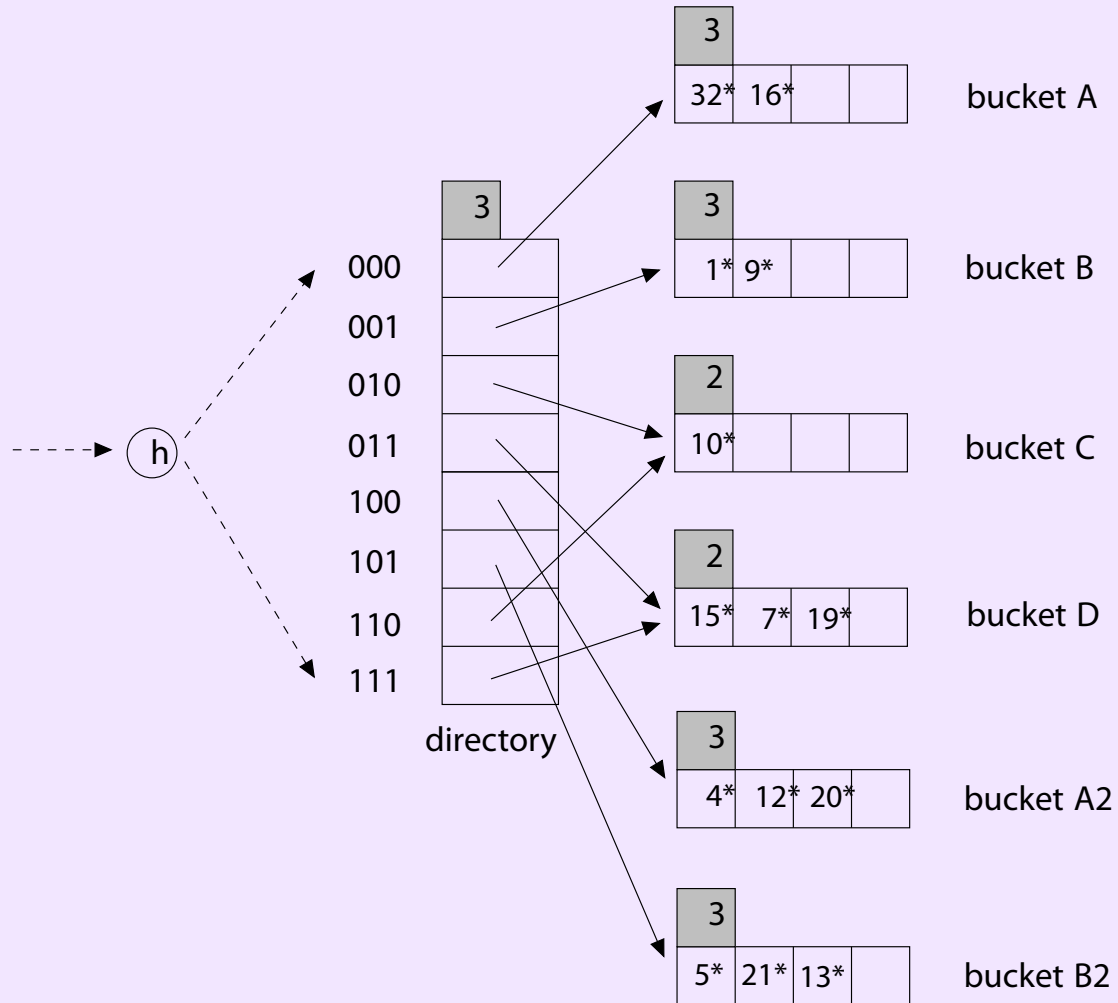
Example (Insert record with $h(k) = 9 = 1001_2$)

- Insert record with key k such that $h(k) = 9 = 1001_2$.
 - The associated bucket B is split, creating a new bucket $B2$. Entries are redistributed. New local depth of B and $B2$ is 3 and thus does *not* exceed the global depth of 3 .
- ⇒ Modifying the directory's bucket pointer for 101_2 is sufficient (see following slide).

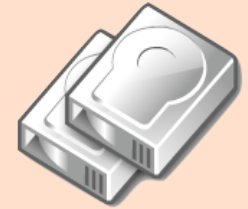


Extendible Hashing: Insert

Example (After insertion of record with $h(k) = 9 = 1001_2$)



Extendible Hashing: Search Procedure

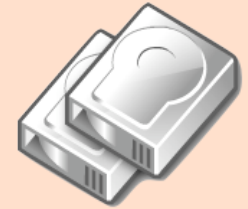


- The following `hsearch(·)` and `hinsert(·)` procedures operate over an in-memory array representation of the bucket directory $bucket[0, \dots, 2^{\boxed{n}} - 1]$.

Extendible Hashing: Search

```
1 Function: hsearch(k)
2 n ←  $\boxed{n}$ ; /* global depth */
3 b ←  $h(k) \& (2^n - 1)$ ; /* mask all but the low n bits */
4 return bucket[b];
```

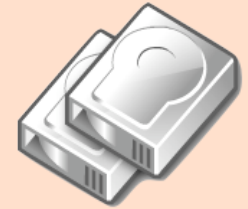
Extendible Hashing: Insert Procedure



Extendible Hashing: Insertion

```
1 Function: hinsert( $k^*$ )
2  $n \leftarrow \boxed{n}$ ;                               /* global depth */
3  $b \leftarrow \text{hsearch}(k)$ ;
4 if  $b$  has capacity then
5   Place  $k^*$  in bucket  $b$ ;
6   return;
7 /* overflow in bucket  $b$ , need to split */
8  $d \leftarrow \boxed{d}_b$ ;                          /* local depth of hash bucket  $b$  */
9 Create a new empty bucket  $b_2$ ;
/* redistribute entries of  $b$  including  $k^*$  */
:
```

Extendible Hashing: Insert Procedure (continued)



Extendible Hashing: Insertion (cont'd)

```
1  ⋮
2  /* redistribute entries of  $b$  including  $k^*$  */
3  foreach  $k^*$  in bucket  $b$  do
4  |   if  $h(k') \& 2^d \neq 0$  then
5  |   |   Move  $k^*$  to bucket  $b_2$  ;
6  |
7  |   /* new local depths for buckets  $b$  and  $b_2$  */
8  |    $d_b \leftarrow d + 1$  ;
9  |    $d_{b_2} \leftarrow d + 1$  ;
10 |   if  $n < d + 1$  then
11 |   |   /* we need to double the directory */
12 |   |   Allocate  $2^n$  new directory entries  $bucket[2^n, \dots, 2^{n+1} - 1]$  ;
13 |   |   Copy  $bucket[0, \dots, 2^n - 1]$  into  $bucket[2^n, \dots, 2^{n+1} - 1]$  ;
14 |   |    $n \leftarrow n + 1$  ;
15 |   |   /* update the bucket directory to point to  $b_2$  */
16 |   |    $bucket[(h(k) \& (2^n - 1)) | 2^n] \leftarrow addr(b_2)$ 
```

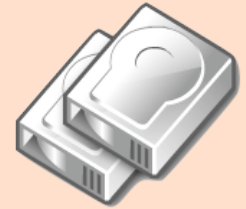
Extendible Hashing: Overflow Chains? / Delete

Overflow chains?

Extendible hashing uses overflow chains hanging off a bucket only as a resort. Under which circumstances will extendible hashing create an overflow chain?

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Overflow chains?

Extendible hashing uses overflow chains hanging off a bucket only as a resort. Under which circumstances will extendible hashing create an overflow chain?

If considering $d + 1$ bits does *not* lead to satisfying record redistribution in procedure `hinsert(k)` (skewed data, hash collisions).

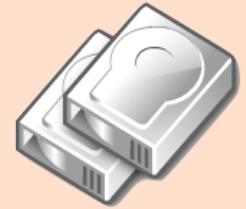
- Deleting an entry k_* from a bucket may leave its bucket completely (or almost) empty.
- Extendible hashing then tries to **merge** the empty bucket and its associated partner bucket.

Extendible hashing: deletion

When is local depth decreased? When is global depth decreased?
(Try to work out the details on your own.)

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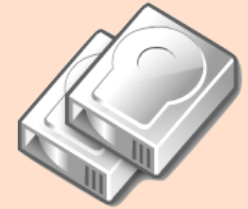
Procedures

Linear Hashing

- **Linear hashing** can, just like extendible hashing, adapt its underlying data structure to record insertions and deletions:
 - Linear hashing **does not need a hash directory** in addition to the actual hash table buckets.
 - Linear hashing can define **flexible criteria that determine when a bucket is to be split**,
 - Linear hashing, however, may perform bad if the key distribution in the data file is *skewed*.
- We will now investigate linear hashing in detail and come back to the points above as we go along.
- The core idea behind linear hashing is to use an **ordered family of hash functions**, h_0, h_1, h_2, \dots (traditionally the subscript is called the hash function's *level*).

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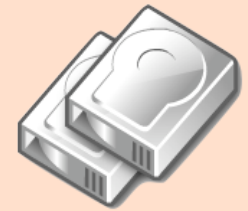
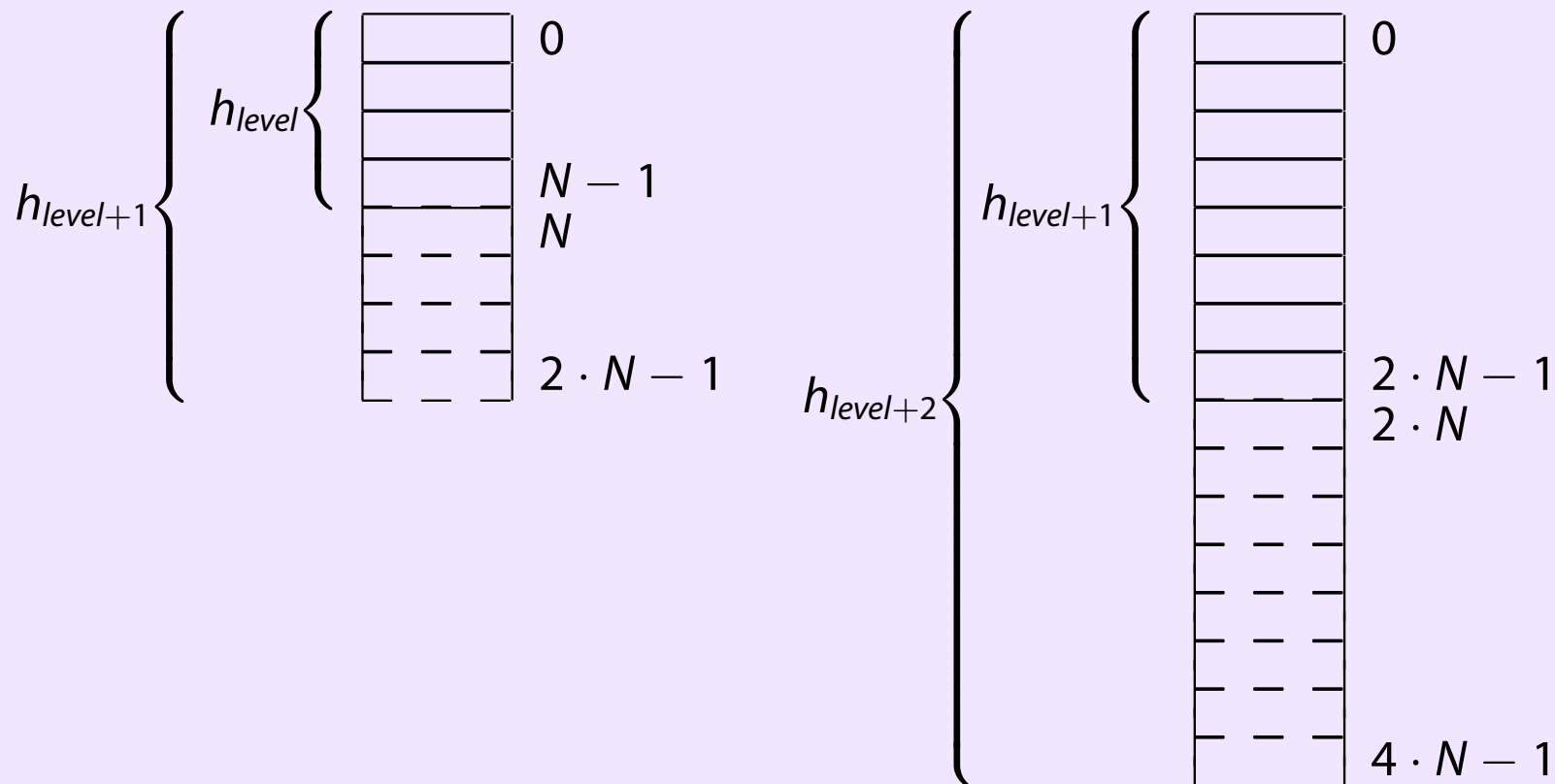
Running Example

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Linear Hashing: Hash Function Family

- We design the family so that the **range of $h_{level+1}$ is twice as large as the range of h_{level}** (for $level = 0, 1, 2, \dots$).

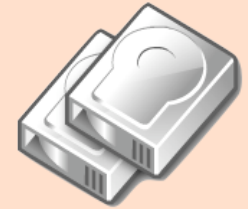
Example (h_{level} with range $[0, \dots, N - 1]$)



Linear Hashing: Hash Function Family

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- Given an initial hash function h and an initial hash table size N , one approach to define such a family of hash functions h_0, h_1, h_2, \dots would be:

Hash function family

$$h_{level}(k) = h(k) \bmod (2^{level} \cdot N) \quad (level = 0, 1, 2, \dots)$$

Linear Hashing: Basic Scheme

Basic linear hashing scheme

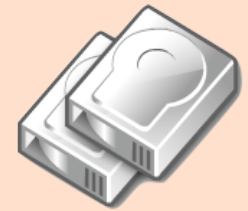
- 1 Initialize: $level \leftarrow 0, next \leftarrow 0$.
- 2 The **current hash function** in use for searches (insertions/deletions) is h_{level} , active hash table buckets are those in h_{level} 's range: $[0, \dots, 2^{level} \cdot N - 1]$.
- 3 **Whenever** we realize that the current **hash table overflows**, e.g.,
 - insertions filled a primary bucket beyond $c\%$ capacity,
 - or the overflow chain of a bucket grew longer than p pages,
 - or *⟨insert your criterion here⟩*

we **split the bucket** at hash table position $next$ (in general, this is **not the bucket which triggered the split!**)



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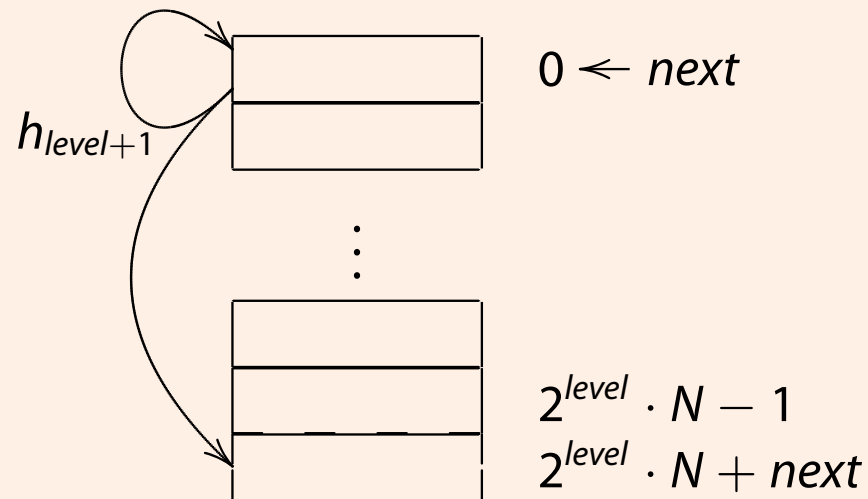
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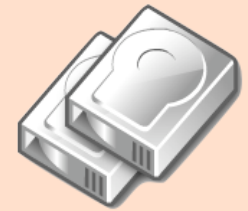
Linear Hashing: Bucket Split

Linear hashing: bucket split

- 1 **Allocate a new bucket, append** it to the hash table (its position will be $2^{level} \cdot N + next$).
- 2 **Redistribute** the entries in bucket *next* by **rehashing** them via $h_{level+1}$ (some entries will remain in bucket *next*, some go to bucket $2^{level} \cdot N + next$). For $next = 0$:



- 3 **Increment** *next* by 1.
- ⇒ All buckets with positions $< next$ have been rehashed.

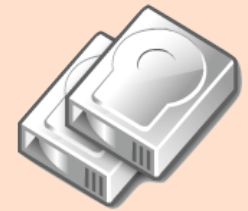
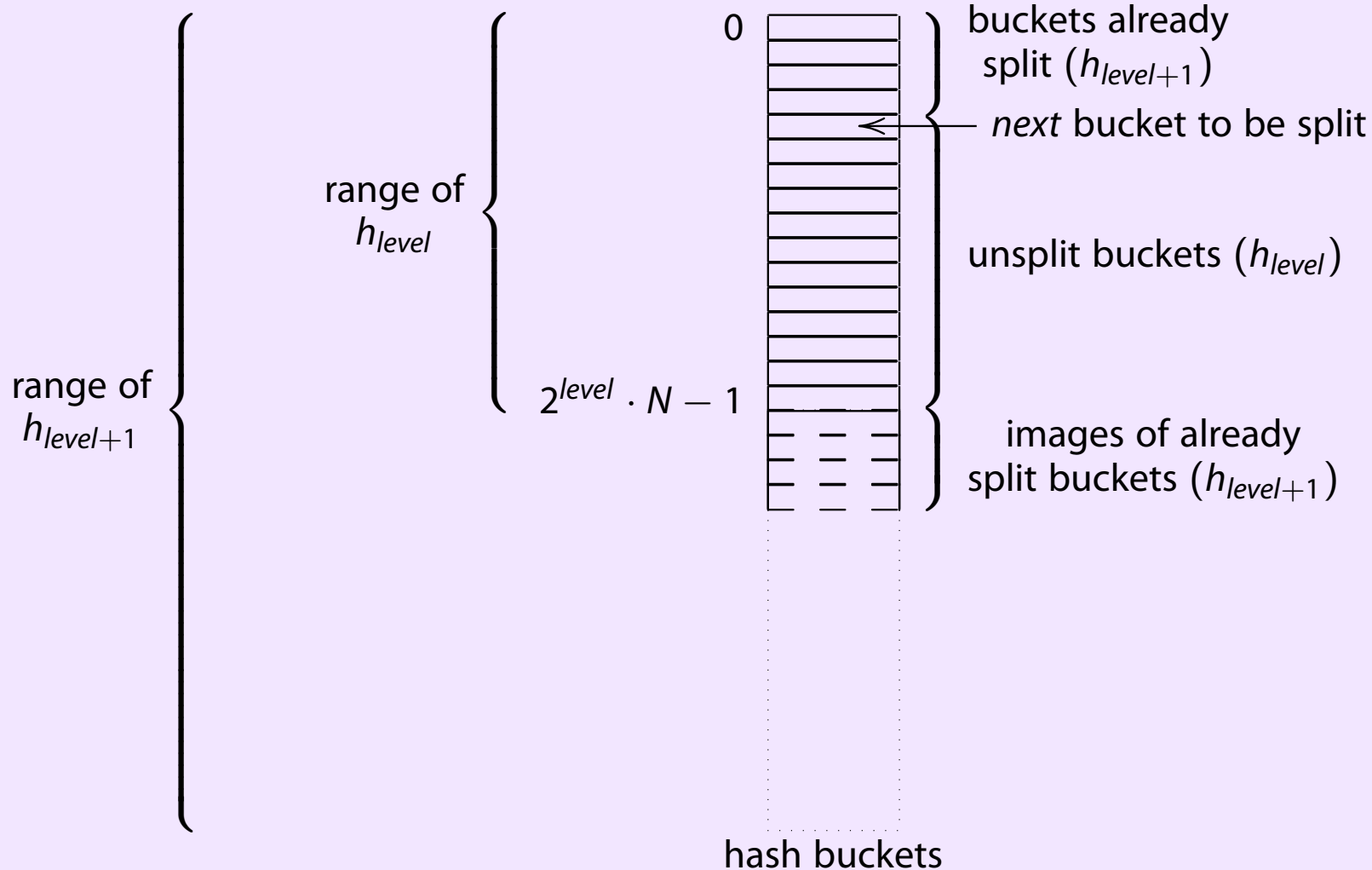


Linear Hashing: Rehashing

Searches need to take current *next* position into account

$$h_{level}(k) \quad \left\{ \begin{array}{l} < next : \text{ we hit an already split bucket, } \mathbf{rehash} \\ \geq next : \text{ we hit a yet unsplit bucket, bucket found} \end{array} \right.$$

Example (Current state of linear hashing scheme)



Linear Hashing: Split Rounds

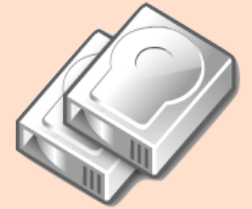
 **When *next* is incremented beyond hash table size...?**

A bucket split increments *next* by 1 to mark the next bucket to be split. How would you propose to handle the situation when *next* is incremented *beyond* the last current hash table position, *i.e.*

$$next > 2^{level} \cdot N - 1?$$

Hash-Based Indexing

Torsten Grust



Hash-Based Indexing

Static Hashing

Hash Functions

Extendible Hashing

Search

Insertion

Procedures

Linear Hashing

Insertion (Split, Rehashing)

Running Example

Procedures

Linear Hashing: Split Rounds

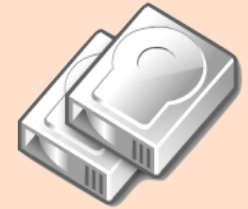
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$$next > 2^{level} \cdot N - 1?$$

Answer:

- If $next > 2^{level} \cdot N - 1$, **all buckets** in the current hash table are hashed via function $h_{level+1}$.
- ⇒ Proceed in a **round-robin fashion**:
If $next > 2^{level} \cdot N - 1$, then
 - ① increment *level* by 1,
 - ② $next \leftarrow 0$ (start splitting from hash table top again).
- In general, an overflowing bucket is *not* split immediately, but—due to round-robin splitting—no later than in the following round.

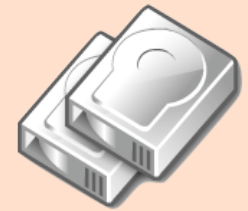
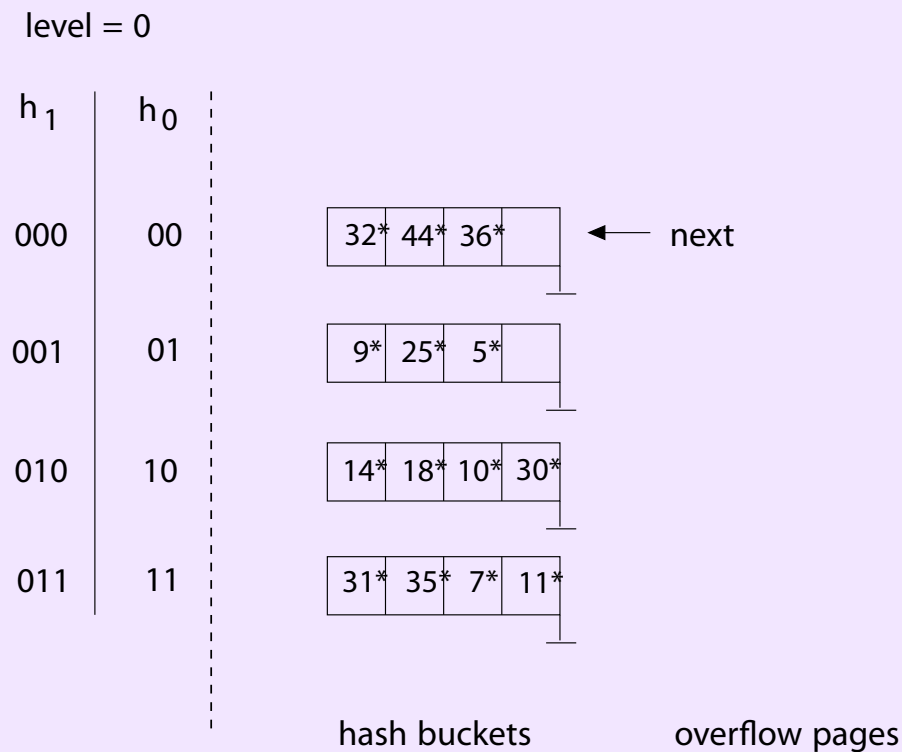


Linear Hashing: Running Example

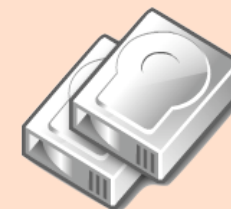
Linear hash table setup:

- Bucket capacity of 4 entries, initial hash table size $N = 4$.
- Split criterion: allocation of a page in an overflow chain.

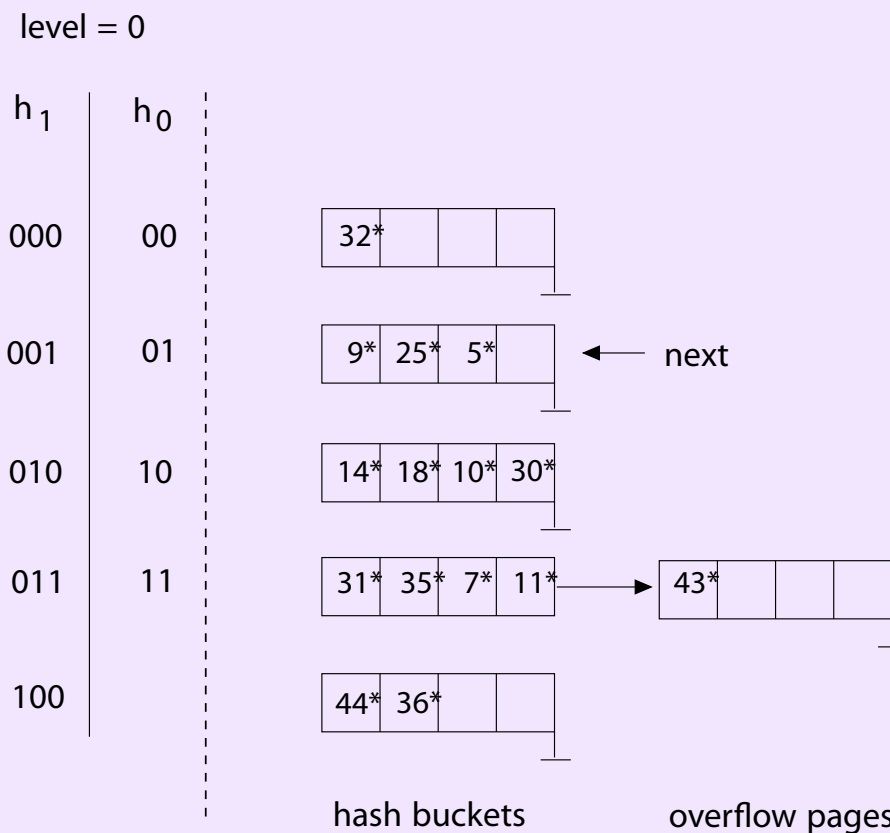
Example (Linear hash table, $h_{level}(k)^*$ shown)



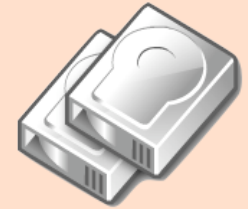
Linear Hashing: Running Example



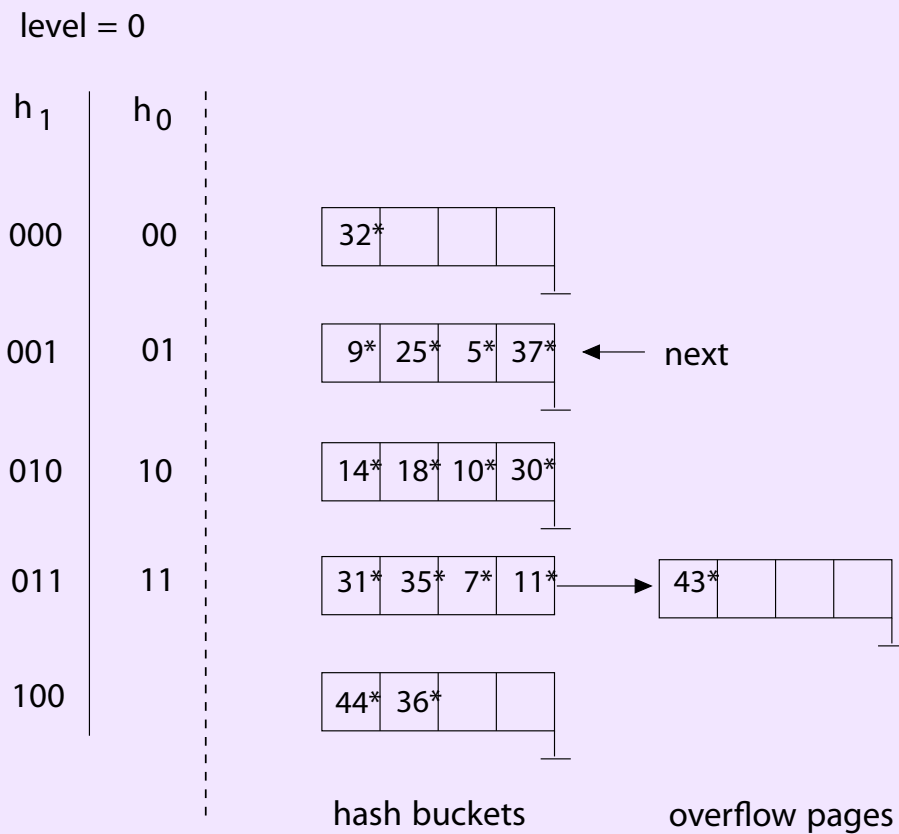
Example (Insert record with key k such that $h_0(k) = 43 = 101011_2$)



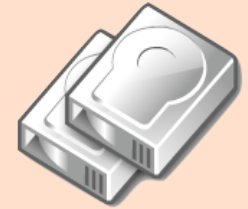
Linear Hashing: Running Example



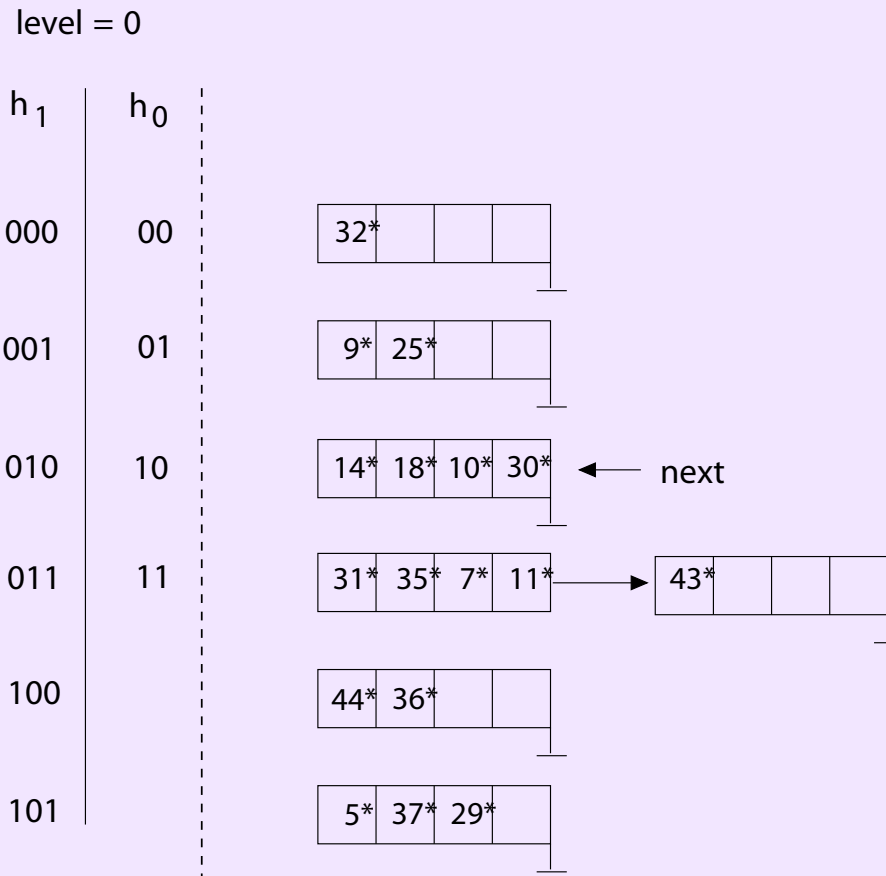
Example (Insert record with key k such that $h_0(k) = 37 = 100101_2$)



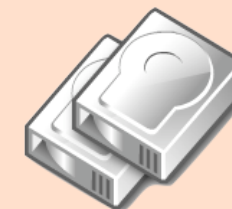
Linear Hashing: Running Example



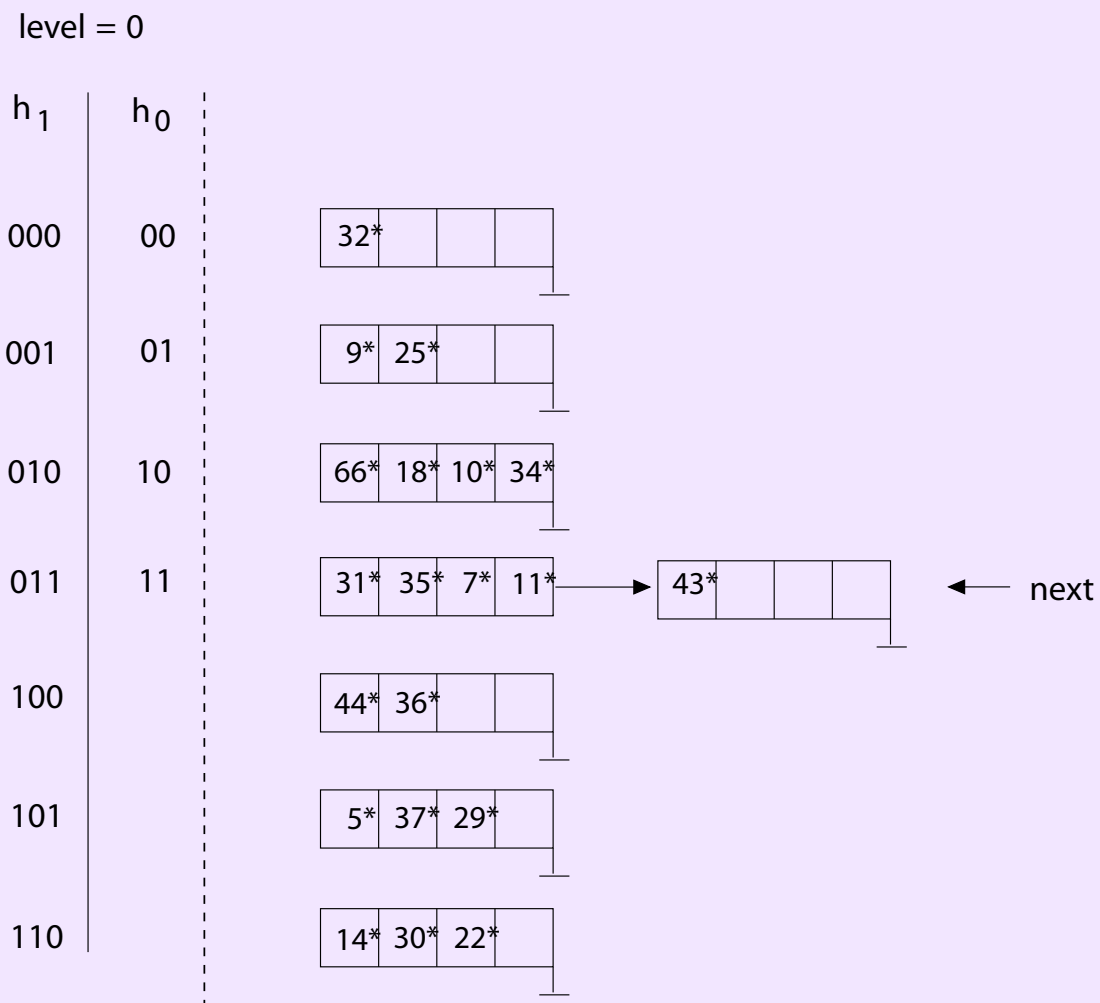
Example (Insert record with key k such that $h_0(k) = 29 = 11101_2$)



Linear Hashing: Running Example

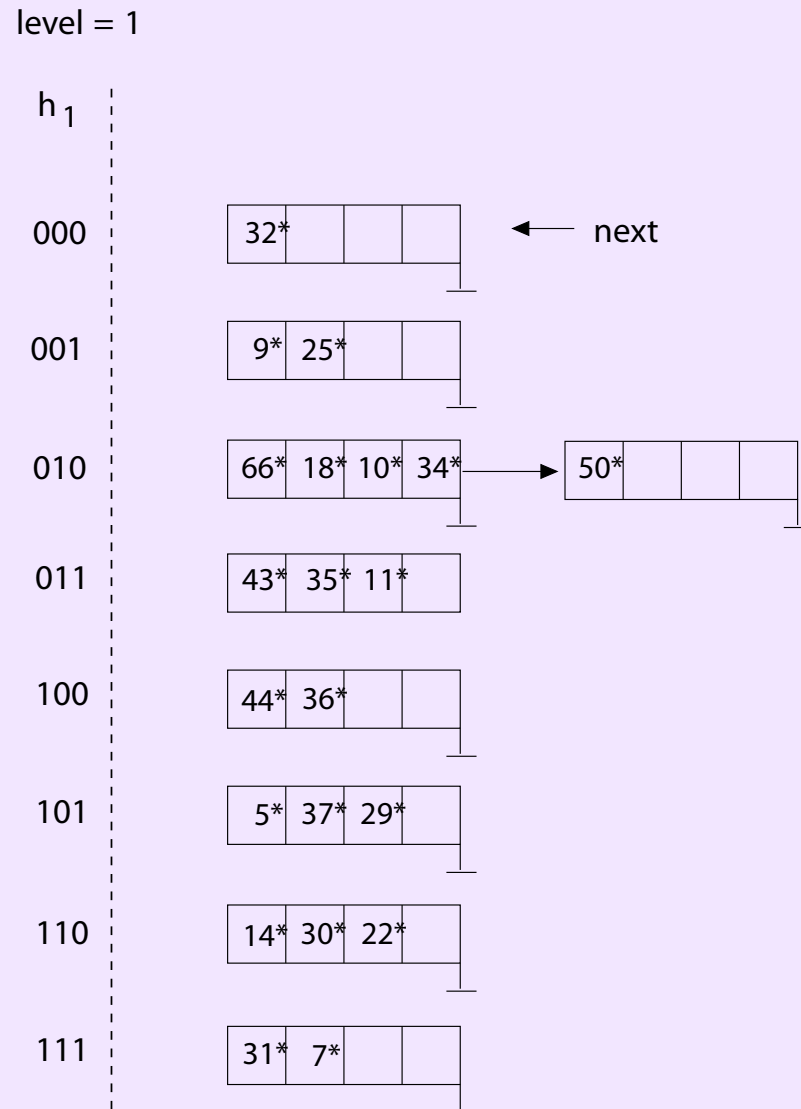


Example (Insert three records with key k such that $h_0(k) = 22 = 10110_2 / 66 = 1000010_2 / 34 = 100010_2$)

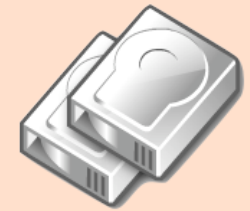


Linear Hashing: Running Example

Example (Insert record with key k such that $h_0(k) = 50 = 110010_2$)



Rehashing a bucket requires rehashing its overflow chain, too.

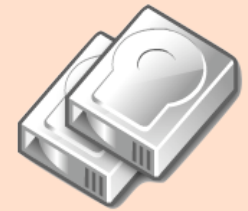


Linear Hashing: Search Procedure

- Procedures operate over hash table bucket (page) address array $bucket[0, \dots, 2^{level} \cdot N - 1]$.
- Variables $level, next$ are hash-table globals, N is constant.

Linear hashing: search

```
1 Function: hsearch( $k$ )  
2  $b \leftarrow h_{level}(k)$  ;  
3 if  $b < next$  then  
   | /*  $b$  has already been split, record for key  $k$  */  
   | /* may be in bucket  $b$  or bucket  $2^{level} \cdot N + b$  */  
   | /*  $\Rightarrow$  rehash */  
4    $b \leftarrow h_{level+1}(k)$  ;  
   | /* return address of bucket at position  $b$  */  
5 return  $bucket[b]$  ;
```

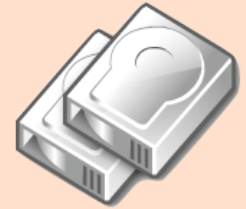


Linear Hashing: Insert Procedure

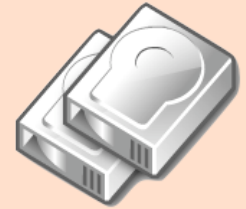
Linear hashing: insert

```
1 Function: hinsert( $k^*$ )
2  $b \leftarrow h_{level}(k)$  ;
3 if  $b < next$  then
4    $\left[ \begin{array}{l} /* rehash \\ b \leftarrow h_{level+1}(k) ; \end{array} \right. */$ 
5 Place  $h(k^*)$  in  $bucket[b]$  ;
6 if  $overflow(bucket[b])$  then
7    $\left[ \begin{array}{l} Allocate\ new\ page\ b' ; \\ /*\ Grow\ hash\ table\ by\ one\ page \\ bucket[2^{level} \cdot N + next] \leftarrow addr(b') ; \\ \vdots \end{array} \right. */$ 
9
```

- Predicate $overflow(\cdot)$ is a tunable parameter: whenever $overflow(bucket[b])$ returns *true*, trigger a split.



Linear Hashing: Insert Procedure (continued)



Linear hashing: insert (cont'd)

```
1  ⋮
2  if overflow(...) then
3      ⋮
4      foreach entry  $k'$  in bucket[next] do
5          /* redistribute                                */
6          Place  $k'$  in bucket[ $h_{level+1}(k')$ ];
7      next  $\leftarrow$  next + 1;
8      /* did we split every bucket in the hash?        */
9      if next >  $2^{level} \cdot N - 1$  then
10         /* hash table size doubled, split from top   */
11         level  $\leftarrow$  level + 1;
12         next  $\leftarrow$  0;
13 return;
```

Linear Hashing: Delete Procedure (Sketch)

- Linear hashing deletion essentially behaves as the “inverse” of `hinsert(·)`:

Linear hashing: delete (sketch)

```
1 Function: hdelete(k)
2   ⋮
3   /* does record deletion leave last bucket empty? */
4   if empty(bucket[2level · N + next]) then
5     Remove page pointed to by bucket[2level · N + next] from hash
6     table ;
7     next ← next - 1 ;
8     if next < 0 then
9       /* round-robin scheme for deletion */
10      level ← level - 1 ;
11      next ← 2level · N - 1 ;
12   ⋮
```

- Possible: replace *empty(·)* by suitable *underflow(·)* predicate.

