DB 2

11 – Sorting and Grouping

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1 ¦ A Family of Q_9: The Ubiquitous Sort

Recall table indexed (with B⁺Tree index indexed_a only):

❶
```
SELECT i.*
FROM   indexed AS i
ORDER BY i.c
```

❷
```
SELECT DISTINCT i.c
FROM   indexed AS i
```

❸
```
SELECT i.c, SUM(i.a) AS s
FROM   indexed AS i
GROUP BY i.c
```

❹
```
SELECT DISTINCT i1.a
FROM   indexed AS i1,
       indexed AS i2
WHERE  i1.a = i2.c :: int
```

All four queries are evaluated using the Sort plan operator.

Sorting Takes Time

- Operator Sort may be costly to evaluate and RDBMSs try to plan query execution without sorting if possible:
  - In queries ❶ to ❹ above, replace i.c (i2.c) by i.a and PostgreSQL will use Index Only Scans on a-ordered B⁺Tree indexed_a instead of Sort.

- Sort is a **blocking operator** and intoduces plan latency:

```
QUERY PLAN
Sort (cost=180530.84..183030.84 rows=1000000 width=41)
 Output: a, b, c
```

Sorting Needs Space

Sorting may need (lots of) **temporary working memory:**

❶ Try to stay RAM-resident if possible,
❷ otherwise, resort to a **disk-based sorting algorithm:**

```
                         QUERY PLAN
 Sort  (cost=180530.84..183030.84 rows=1000000 …) (actual time=…)
 ❶ Sort Method: quicksort  Memory: 102702kB               ◀┐
   Buffers: shared hit=9343                                │
                                                           or
                                                           │
 ❷ Sort Method: external sort  Disk: 50880kB              ◀┘
   Buffers: shared hit=9343, temp read=6360 written=6360
```

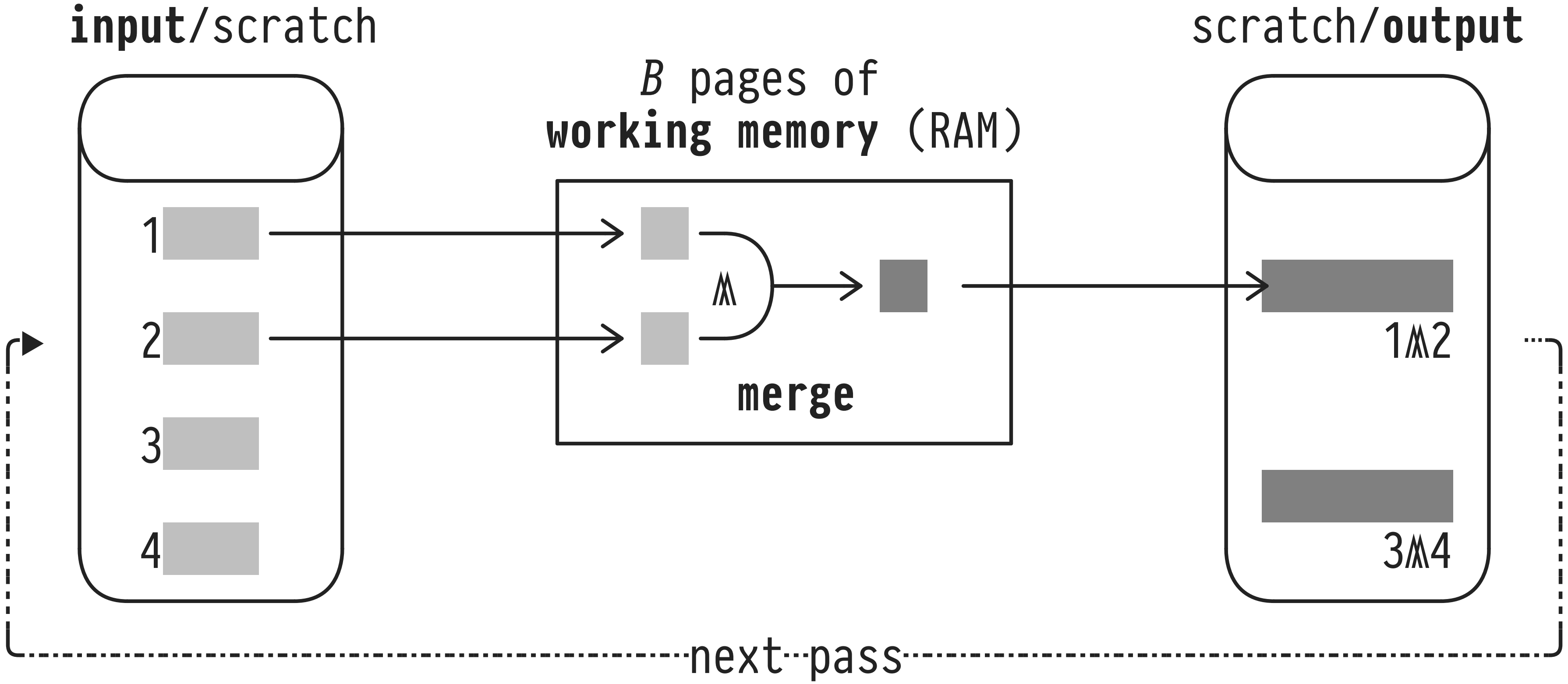
Now assume the following typical scenario:

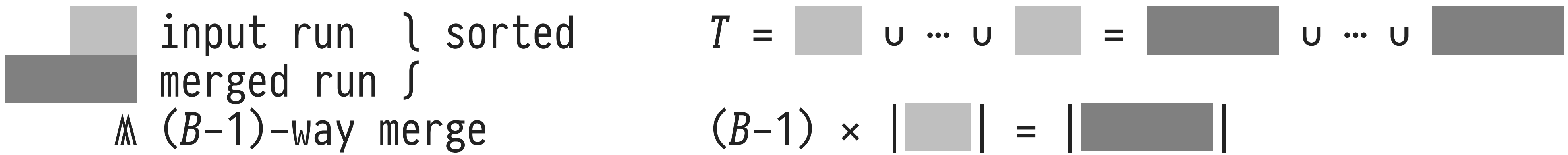
- input heap file T to be sorted: N pages,
- size of temporary working memory (RAM): $B \ll N$ pages,
- size of secondary scratch memory (disk): $\geqslant 2 \times N$ blocks.

External Merge Sort can sort heap files of any size as long as $B \geqslant 3$ pages of working memory are available:

- reads unsorted input file, writes sorted output file,
- creates partially sorted sub-files (*runs*) on disk,
- multiple passes (the larger B, the fewer passes).

An External Merge Sort Pass (B = 3)





External Merge Sort

```
ExternalMergeSort(T,B):
  N ← #pages of T;
  R ← ⌈N/B⌉;                            } R: current number of runs

  split input T into R partitions pᵢ of B pages;  ⎫
  for each i ∈ 1…R                                  ⎬ pass 0
   ⌊ run rᵢ ← in-memory sort of pᵢ;                ⎭

  while R > 1
   | R ← ⌈R / (B-1)⌉;                   ⎫
   | for each i ∈ 1…R                   ⎬ passes 1,2,…
   ⌊  ⌊ ⩕: merge next B-1 runs into one run;  ⎭

  return single sorted run;
```

External Merge Sort: Passes and I/O Operations

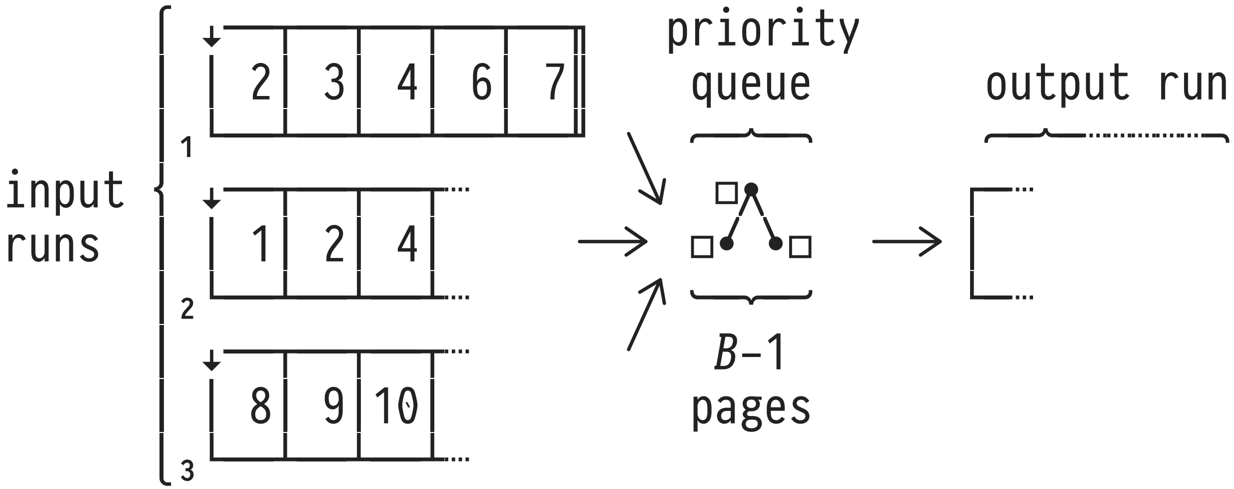
pass	input: #runs	input: run size	output: #runs	output: run size
1	$\lceil N/B \rceil$	B	$\lceil N/B \rceil$ / $(B-1)$	$B \times (B-1)$
2	$\lceil N/B \rceil$ / $(B-1)$	$B \times (B-1)$	$\lceil N/B \rceil$ / $(B-1)^2$	$B \times (B-1)^2$
3	$\lceil N/B \rceil$ / $(B-1)^2$	$B \times (B-1)^2$	$\lceil N/B \rceil$ / $(B-1)^3$	$B \times (B-1)^3$
⋮				
n	$\lceil N/B \rceil$ / $(B-1)^{n-1}$	$B \times (B-1)^{n-1}$	$\lceil N/B \rceil$ / $(B-1)^n$	$B \times (B-1)^n$

- In each pass:
  N = input (#runs × run size) = output (#runs × run size).
  - Each pass performs $2 \times N$ I/O operations.

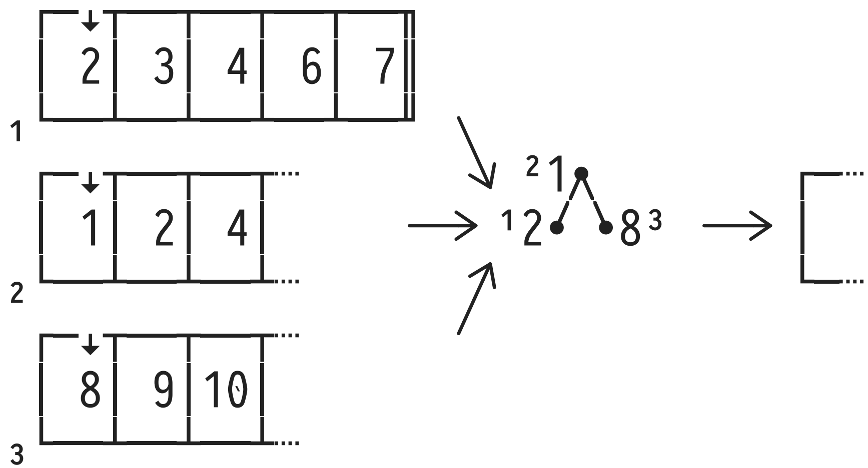
- Passes required by External Merge Sort with B buffers:

$$\underbrace{1}_{\text{pass 0}} + \underbrace{\lceil \log_{B-1} \lceil N/B \rceil \rceil}_{\text{merge passes}}$$

3 ¦ (B–1)-Way Merge (Passes 1,2,…)

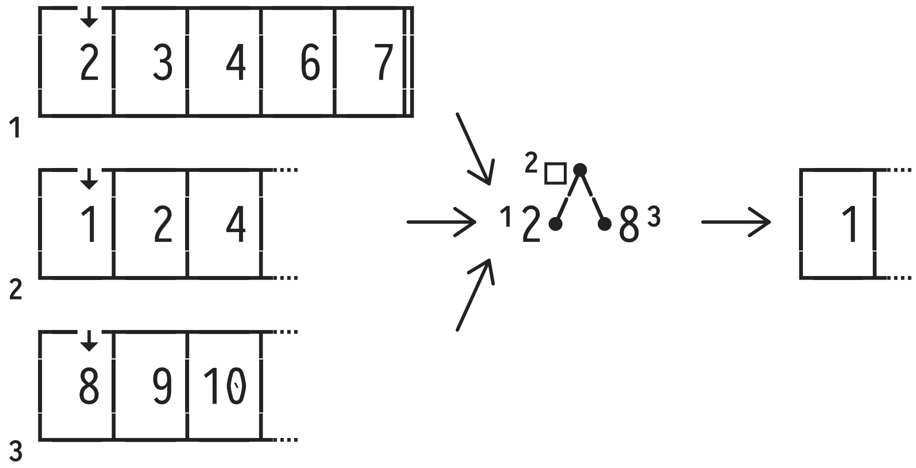


initial merge state

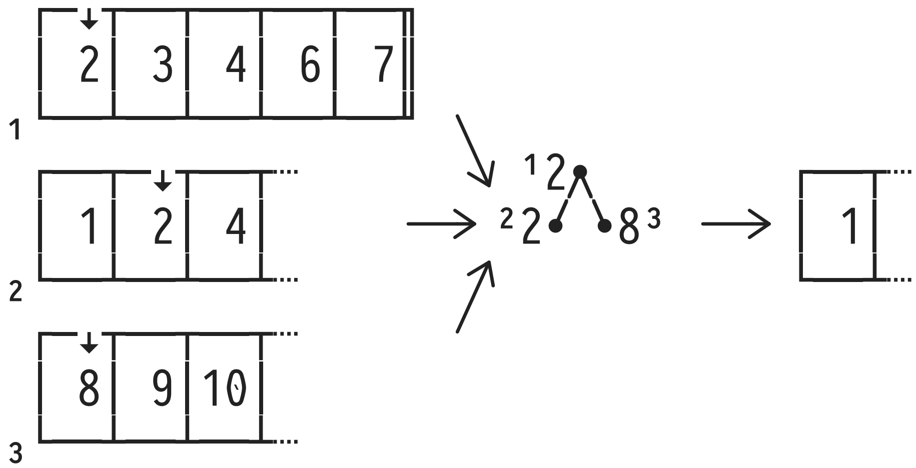


load queue,
entries from input run r_i
are tagged $\square^i$

(B-1)-Way Merge (Passes 1,2,…)

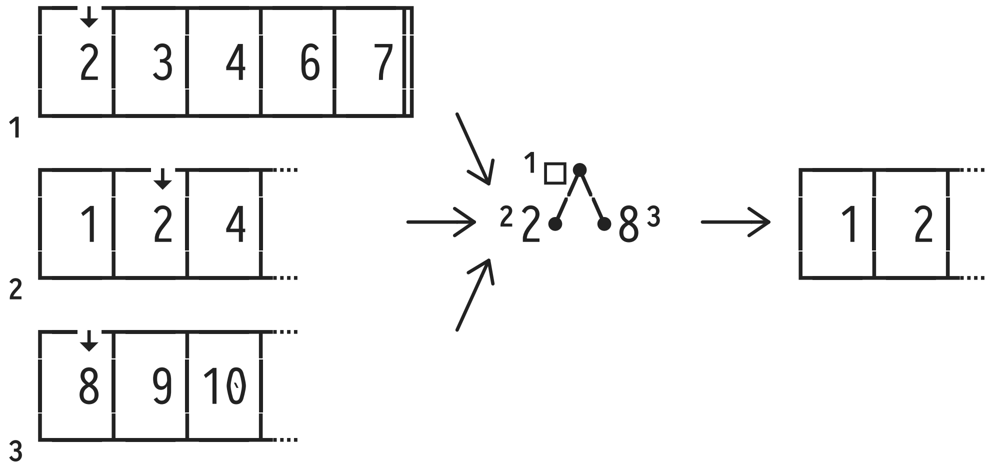


queue head → output run

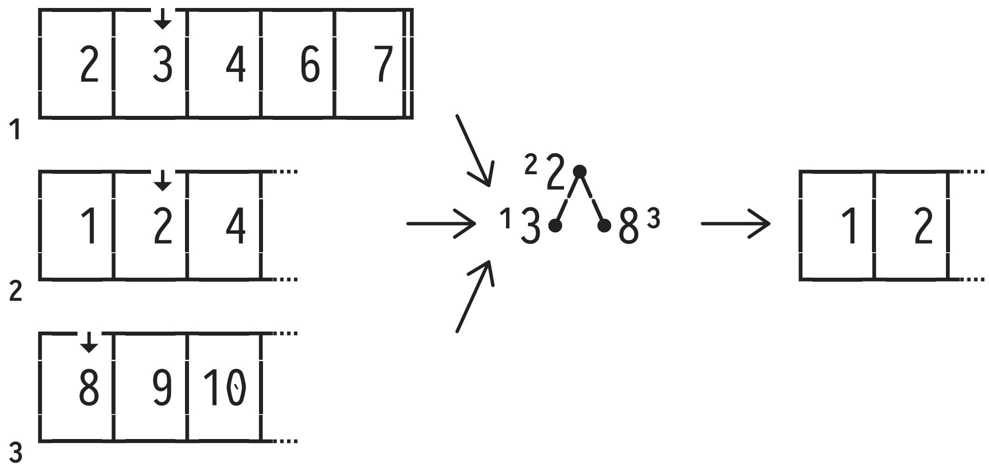


refill queue from input run

(B-1)-Way Merge (Passes 1,2,…)



queue head → output run



refill queue from input run

⋮

External Merge Sort: Access Patterns and Blocked I/O

- I/O access patterns in
  - pass 0: sequential read/write chunks of B pages, 👍
  - merge passes 1,...: random reads from the $B-1$ runs. 👎

- 💡 Perform **blocked I/O** in merge passes 1,2,...:
  - Seek once to read $b > 1$ pages at a time from each run. Reduces per-page I/O cost by a factor of $\approx b$.
  - Reduced fan-in: can only merge $\lfloor (B-1)/b \rfloor$ runs per pass.

External Merge Sort Parameters (Interactive)

I/O Characteristics and Performance of External Sorting

Database Characteristics

Database page size: 8 KiB
Available working space in database buffer (*B*): 16384 pages (that's 128.0 MiB)
I/O blocking factor (*b*): 64 pages

Disk Characteristics

Disk seek time: 3.4 ms
Disk read/write speed: 163 MiB/s
Resulting transfer time for a 8 KiB block: 0.049 ms

Size of Sort Problem

Size of input file to be sorted: 0.5 GiB (this makes for *N* = 65536 pages of input)

Resulting External Sort Behavior

Pass 0 will produce 4 runs, each of size 16384 pages .
We will need 1 merge passes, with a fan-in of 255.

Resulting I/O and Disk Seek Effort

The sort process will initiate 262144 I/O operations (reads and writes) and 2056 disk head seeks.

Resulting Overall Time for Sort Process

Disk seeking will need 0.1 minutes, while 0.2 minutes is spent on I/O itself.
Overall, we end up waiting **0.3 minutes** for the sort result.

Made with Tangle.js.

Interactive

4 ¦ Pass 0: Reducing the Number of Runs

- The *initial number of runs created in pass 0* influence overall sort performance:

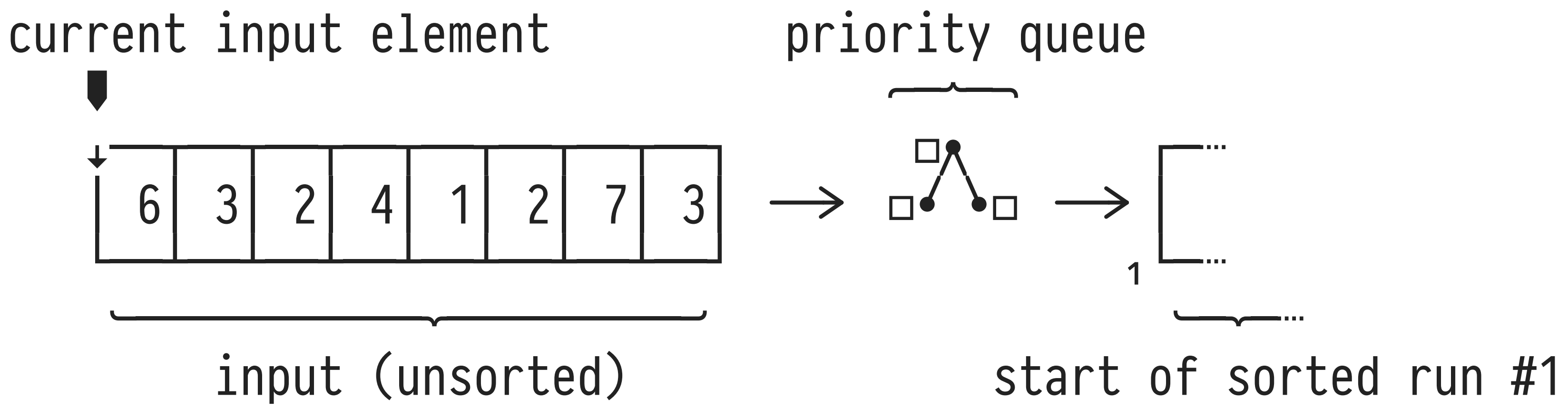
$$\# \text{ I/O operations} = 2 \times N \times (1 + \lceil \log_{B-1} \underbrace{\lceil N/B \rceil}_{\#\text{ runs created in pass 0}} \rceil)$$

- **Q:** Given only B buffers, can we create sorted runs *longer than* B pages?
  - **A:** Yes! In pass 0, use **Replacement Sort** (instead of QuickSort, for example).

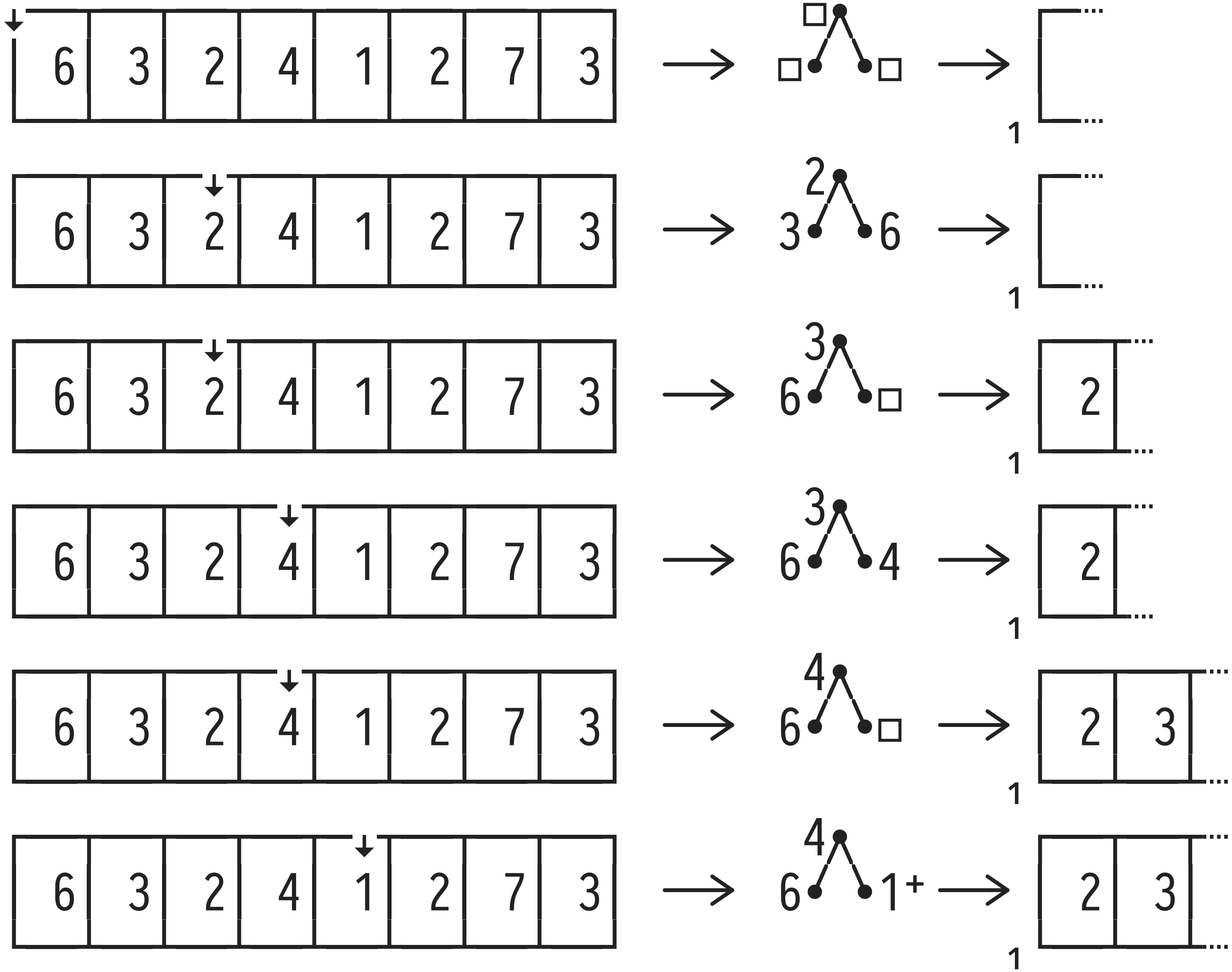
Replacement Sort

Again, use $B-1$ buffer pages to set up a **priority queue:**

1. Elements arriving too late for inclusion in current run are marked (□⁺) and receive lower priority.
2. When all elements in queue are marked, close the current run, unmark all elements, open a new run.

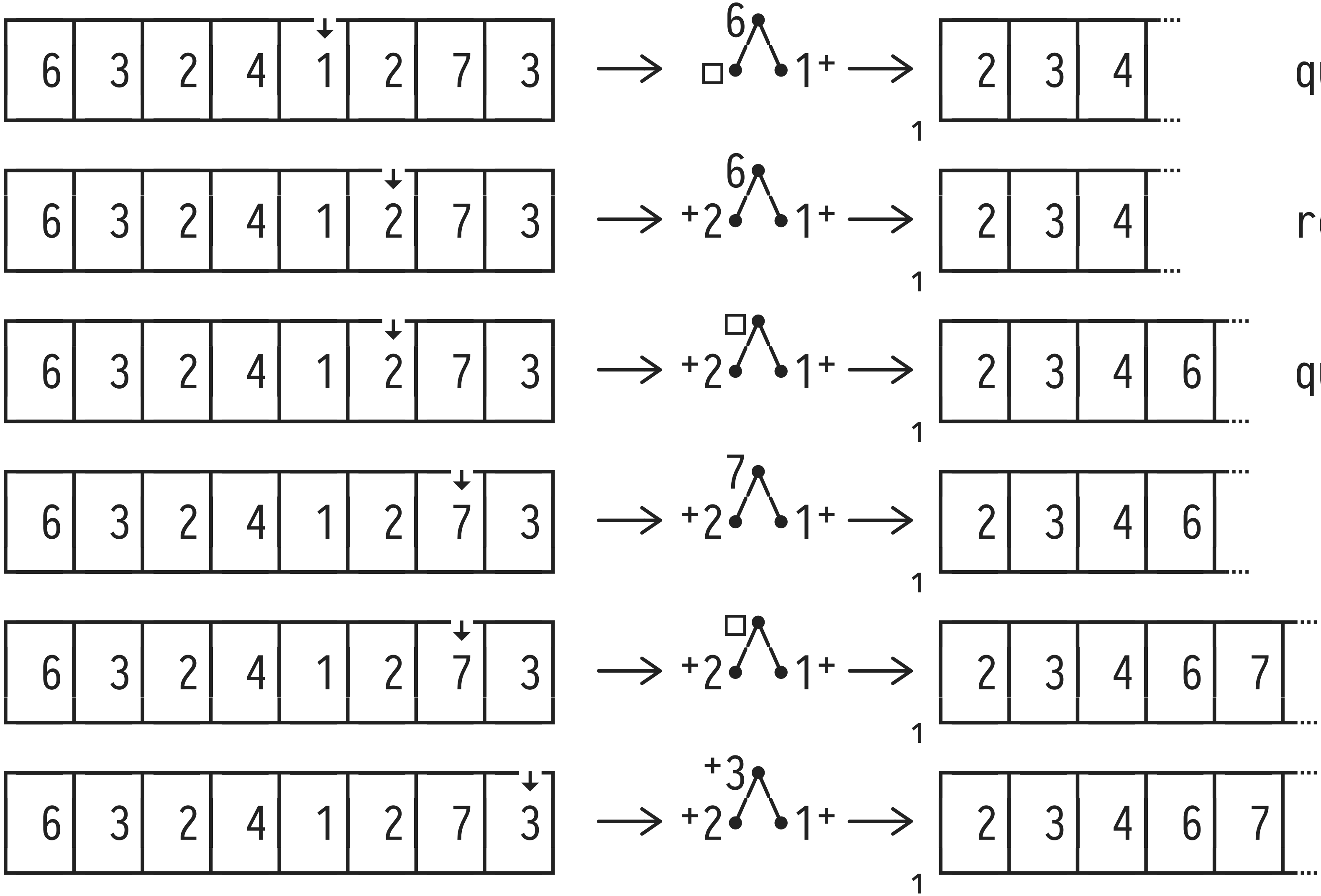


Replacement Sort ($B = 4$)



- initial state
- load queue
- queue head → run #1
- refill queue from input
- queue head → run
- refill q, 1 is late (□⁺)

Replacement Sort ($B = 4$)



queue head → run

refill q, 2 late

queue head → run

refill queue

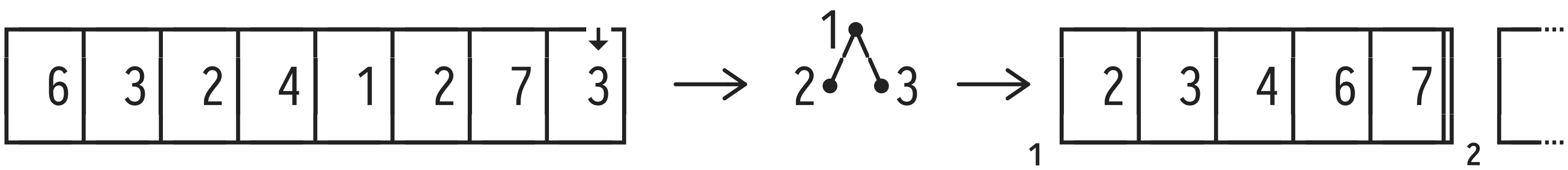
q head → run

refill, 3 late

Replacement Sort (B = 4)

All entries in queue are late (□⁺):

- Close current run #1, open new run #2.
- Reorder entries in queue, continue processing.



- Replacement Sort produces runs of length $\approx 2 \times (B-1) > B$ (see Knuth, TAoCP, volume 3, p. 254).
- Replacement Sort generates longer runs if input file is almost sorted (*e.g.*, consider a heap file that was once clustered but has received a few updates since then).

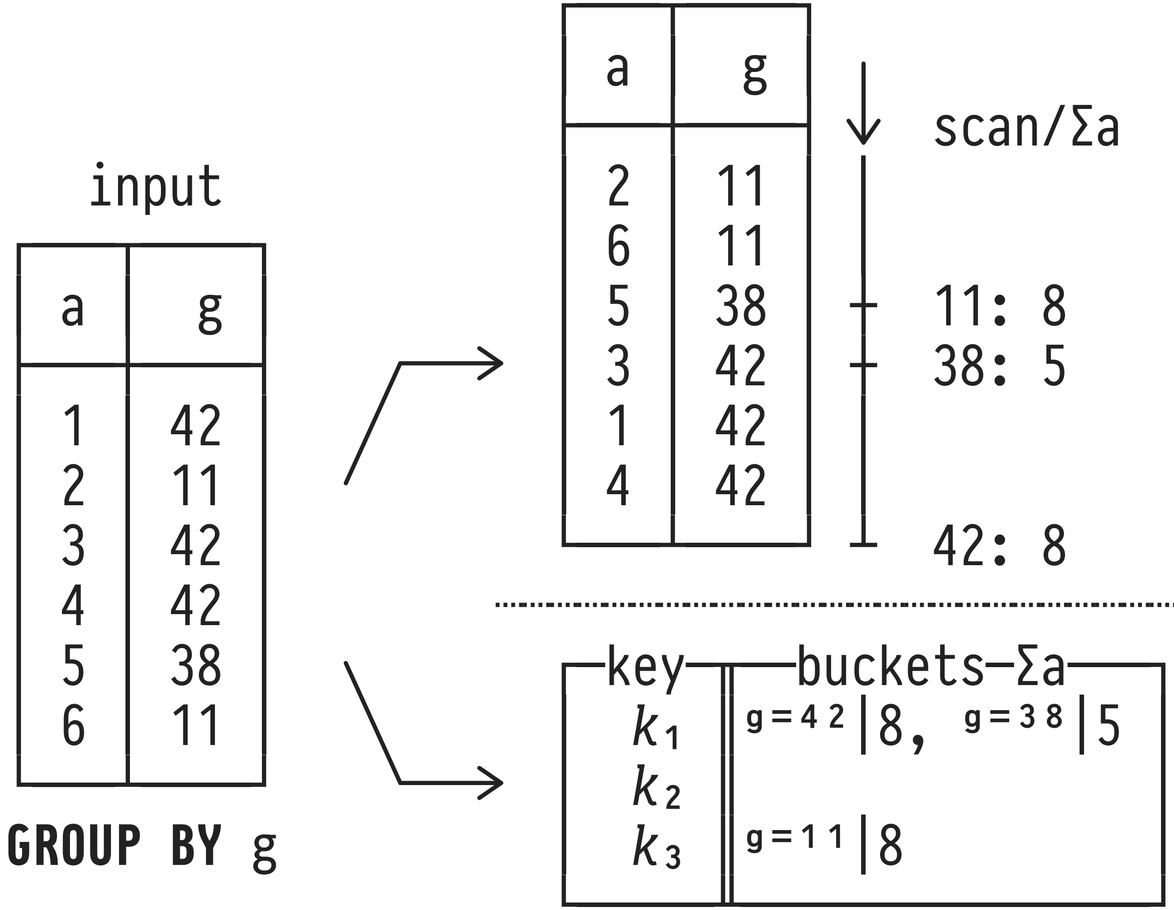
Grouping coarsens the granularity of data processing (individual rows ↘ groups of rows):

```
❷ SELECT g.g, SUM(g.a) AS s -- out: 10⁴ groups (aggregates)
  FROM   grouped AS g       -- in:  10⁶ rows
❶ GROUP BY g.g
```

❶ **Partition** table indexed by criterion g.g (all rows agreeing on g.g form one group),

❷ output group criterion and **aggregates** of the group's member rows (the group member rows themselves are never output).

Grouping: Sorting vs. Hashing



- scan sorted table for group boundaries
- aggregate while scanning

Sorting

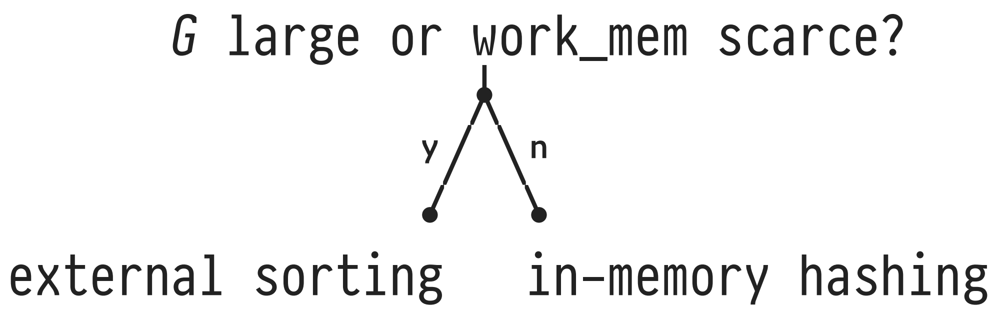
Hashing

- hash buckets hold grouping criterion and aggregate value

Grouping: Sorting vs. Hashing

PostgreSQL plans for sorting vs. hashing based on

1. the available working memory (work_mem) and
2. the estimated number *G* of resulting groups:



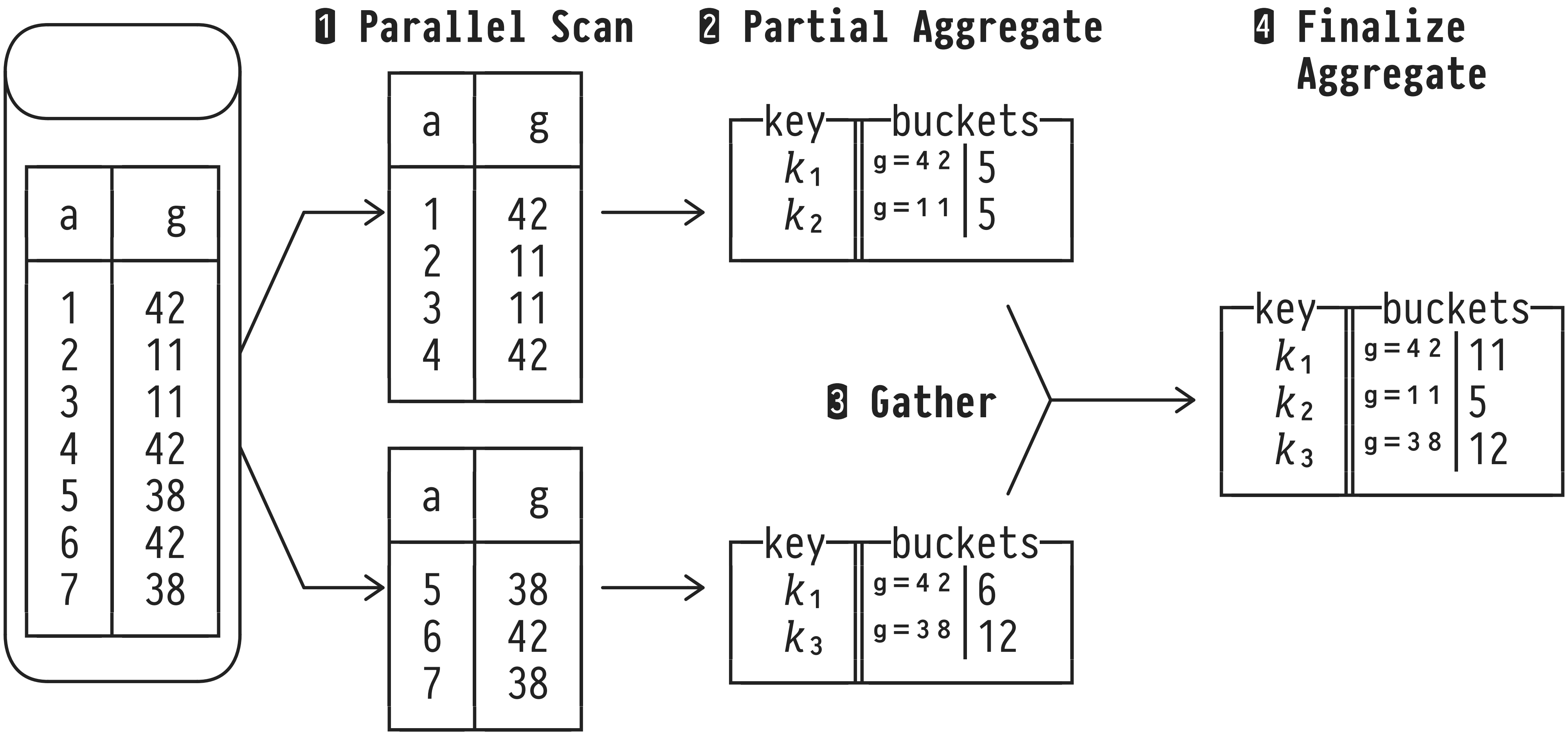
- Often, *G* is unknown or cannot be derived (*e.g.*, GROUP BY g.g % 2 ⇒ $G \leqslant 2$ not understood by PostgreSQL).
  - ⇒ Overestimate *G* conservatively, use sorting.

6 ¦ Parallel Grouping and Aggregation

Grouping and aggregation are query operations that are straightforward to **parallelize:**

- Spawn **workers,** each of which execute in ∥ (on dedicated CPU core). Constrain max number of workers to fit host.
- Try to **evenly distribute work** (*e.g.*, data volume) among workers.
- Assign a **leader** thread/process that coordinates workers and **gathers partial query results.**
- After gathering, **merge/finalize partial results** to produce a single complete query result.

Parallel Grouping (GROUP BY g — SUM(a))



Parallel Grouping for Q_{10}

```
EXPLAIN
  SELECT g.g, SUM(g.a) AS s
  FROM   grouped AS g
  GROUP BY g.g;
```

QUERY PLAN
Finalize HashAggregate (cost=13869.28..13969.02 …)
Group Key: g
-> **Gather** (cost=11675.00..13769.54 …)
Workers Planned: 2 ⬅ *‖ism degree*: 3 (2 *worker* + 1 *leader*)
-> **Partial HashAggregate** (cost=10675.00..10774.74 …)
Group Key: g
-> **Parallel Seq Scan** on grouped g (cost=0.00..8591.67 …)

Partial Aggregation and Finalization

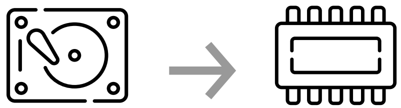
- Parallel evaluation of aggregate *AGG* depends on the **distributivity** over ⊎ (bag union):

$$AGG(X \uplus Y) = AGG(\{AGG(X)\} \uplus \{AGG(Y)\}).$$

- Many SQL aggregates (COUNT, SUM, MAX, MIN, AVG, bool_and, bool_or, …) exhibit this property:

$$\text{SUM}(\underbrace{X \uplus Y}_{\text{distribute work}}) = \text{SUM}(\underbrace{\{\text{SUM}(X)\}}_{\text{partial}} \uplus \underbrace{\{\text{SUM}(Y)\}}_{\text{aggregates}}) = \text{SUM}(X) \underset{\text{finalize}}{\underset{\uparrow}{+}} \text{SUM}(Y)$$

7 ¦ Q_9: Sorting in MonetDB



```
CREATE TABLE sorted (a text, s int);
  ⋮
SELECT s.a, s.s
FROM   sorted AS s
ORDER BY s.s [, s.a]    -- single- or multi-column criteria
```

MonetDB's BATs already provide **ordered row storage.** Some ORDER BY queries will thus be no-ops (recall tail properties sorted, revsorted).

Otherwise, use **order indexes**—either persistent or computed on the fly—to apply column re-ordering.

Recall: Order Indexes (ORDER BY s.s)

a

head	tail
0@0	a
1@0	b
2@0	c
3@0	d
4@0	e
5@0	f
6@0	g
7@0	h
8@0	i
9@0	j

s

tail
40
0
50
30
50
10
50
10
10
20

$oidx^{s}$

head	tail
0@0	1@0
1@0	7@0
2@0	8@0
3@0	5@0
4@0	9@0
5@0	3@0
6@0	0@0
7@0	2@0
8@0	6@0
9@0	4@0

$a^{ord(s)}$

head	tail
0@0	b
1@0	h
2@0	i
3@0	f
4@0	j
5@0	d
6@0	a
7@0	c
8@0	g
9@0	e

$s^{ord(s)}$

tail
0
10
10
10
20
30
40
50
50
50

…**algebra.**
projection($oidx^{s}$,·)

Order Indexes on the Fly: algebra.sort

```
EXPLAIN
  SELECT s.a, s.s
  FROM   sorted AS s
  ORDER BY s.s;

sorted :bat[:oid] := sql.tid(sql, "sys", "sorted");
s0     :bat[:int] := sql.bind(sql, "sys", "sorted", "s", …);
s      :bat[:int] := algebra.projection(sorted, s0);
(s^ord(s), oidx^s, gidx^s)              desc↴      ↙stable
                  := algebra.sort(s, false, false);
a0     :bat[:str] := sql.bind(sql, "sys", "sorted", "a", …);
a      :bat[:str] := algebra.projection(sorted, a0);
a^ord(s):bat[:str] := algebra.projectionpath(oidx^s, sorted, a0);

io.print(a^ord(s), s^ord(s));
```

Persistent Order Indexes

If sorting is central to the query workload, create a **persistent order index** that is immediately applicable:

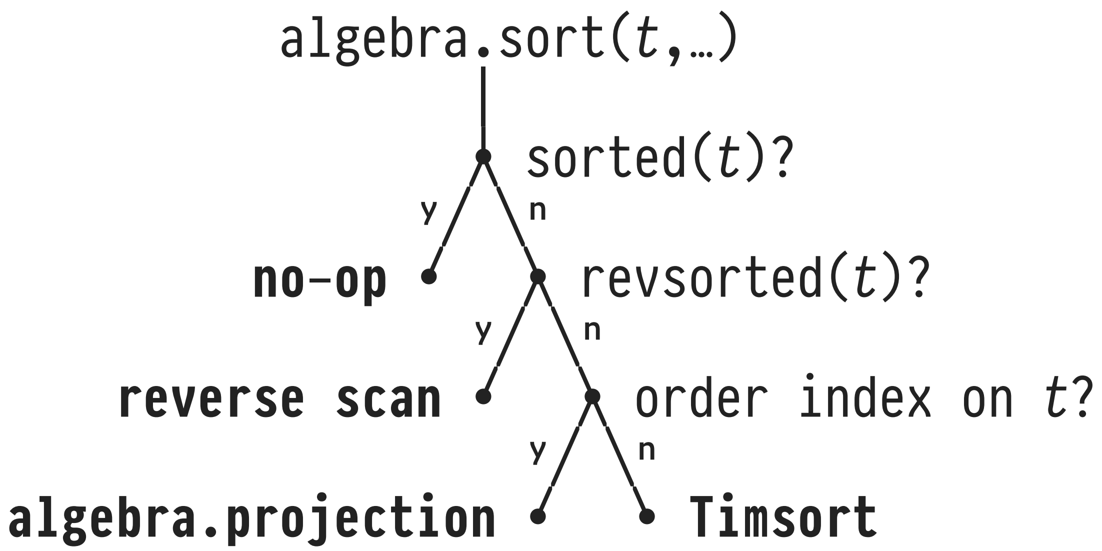
```
ALTER TABLE sorted SET READ ONLY;      -- ⚠️

CREATE ORDERED INDEX oidxˢ ON sorted(s);
```

- Order indexes are **static** structures that are *not* dynamically maintained (as opposed to B⁺Trees).
  If order index has been created...
  1. on the fly: throw away on table update,
  2. persistent: read-only table, no updates at all.

Tactical Optimization for algebra.sort

- algebra.sort aims to avoid actual sorting effort based on properties of BAT *t* and the presence of order indexes:



- If all else fails, apply in-memory sort algorithm **Timsort** (1993; hybrid of merge/insertion sort, run-aware).

8 ┆ Multi-Criteria ORDER BY

Multi-column ordering criteria require special treatment: algebra.sort(s) only receives single criterion s.

```
SELECT s.a, s.s
FROM   sorted AS s
ORDER BY s.s, s.a -- s₁ < s₂ ⇔ s₁.s < s₂.s ∨
                  --            (s₁.s = s₂.s ∧ s₁.a < s₂.a)
```

- 💡 Let algebra.sort(s) return *three* result BATs:
  1. $s^{ord(s)}$ (the ordered input s) ✓
  2. $oidx^{s}$ (order index) ✓
  3. $gidx^{s}$ (groups rows that agree on criterion s).

Multi-Criteria ORDER BY: Group Index **gidx**

$s^{ord(s)}$ ✓ , $oidx^{s}$ ✓ , $\mathbf{gidx}^{s}$:= algebra.sort(s)

$s^{ord(s)}$ ✓

head	tail
0@0	0
1@0	10
2@0	10
3@0	10
4@0	20
5@0	30
6@0	40
7@0	50
8@0	50
9@0	50

$oidx^{s}$ ✓

head	tail
0@0	1@0
1@0	7@0
2@0	8@0
3@0	5@0
4@0	9@0
5@0	3@0
6@0	0@0
7@0	2@0
8@0	6@0
9@0	4@0

$\mathbf{gidx}^{s}$

head	tail	s =
0@0	0@0	0
1@0	1@0	10
2@0	1@0	10
3@0	1@0	10
4@0	2@0	20
5@0	3@0	30
6@0	4@0	40
7@0	5@0	50
8@0	5@0	50
9@0	5@0	50

3 output BATs

s

head	tail
0@0	40
1@0	0
2@0	50
3@0	30
4@0	50
5@0	10
6@0	50
7@0	10
8@0	10
9@0	20

input BAT

Multi-Criteria ORDER BY s,a: Refine ORDER BY s by a

algebra.sort(s)
⋮
→ $s^{ord(s)}$
→ $oidx^{s}$
→ $gidx^{s}$

$gidx^{s}$

head	tail
0@0	0@0
1@0	1@0
2@0	1@0
3@0	1@0
4@0	2@0
5@0	3@0
6@0	4@0
7@0	5@0
8@0	5@0
9@0	5@0

$a^{ord(s)}$

head	tail	exec in ∥
0@0	b	↕ *sort*
1@0	h	↑
2@0	i	*sort*
3@0	f	↓
4@0	j	↕ *sort*
5@0	d	↕ *sort*
6@0	a	↕ *sort*
7@0	c	↑
8@0	g	*sort*
9@0	e	↓

$refine^{s\ by\ a}$

head	tail
0@0	0@0
1@0	3@0
2@0	1@0
3@0	2@0
4@0	4@0
5@0	5@0
6@0	6@0
7@0	7@0
8@0	9@0
9@0	8@0

…**algebra.sort***($a^{ord(s)}$,$gidx^{s}$)

- Apply this to $c^{ord(s)}$ to obtain $c^{ord(sa)}$.
- Apply this to $oidx^{s}$ to obtain $oidx^{sa}$.