Advanced SQL

04 — Window Functions

Torsten Grust
Universität Tübingen, Germany

1 Window Functions

With SQL:2003, the ISO SQL Standard introduced **window functions**, a new mode of row-based computation:

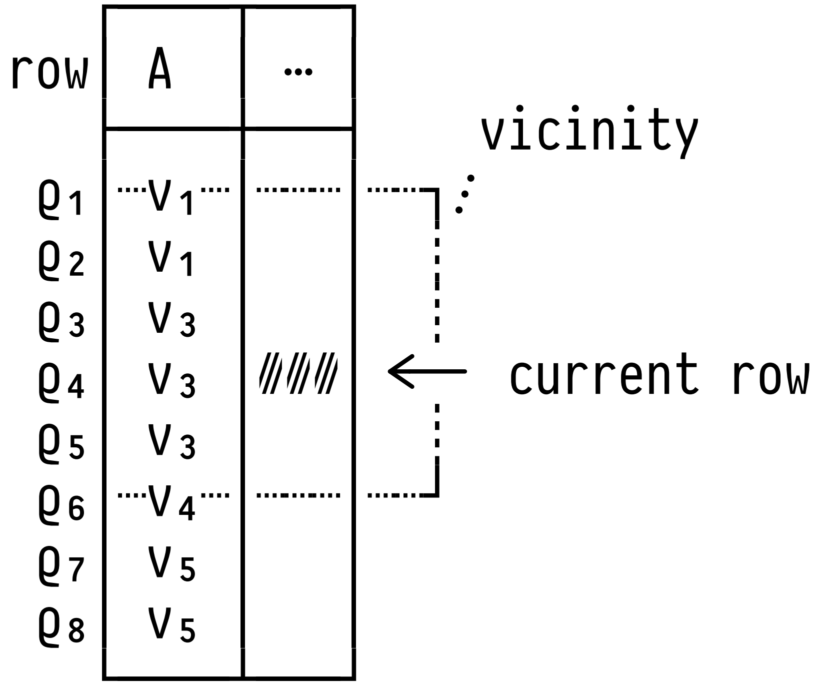
SQL Modes of Computation

SQL Feature	Mode of Computation
function	row → row
table-generating function	row → table of rows
aggregate	group of rows → row (one per group)
window function ☜	row vicinity → row (one per row)

Window functions …

- … are **row-based:** each individual input row r is mapped to one result row,
- … use the **vicinity** around r to compute this result row.

Row Vicinity: Window Frames



- Each row is the **current row** at one point in time.

- Row vicinity (**window**, **frame**) is based on either:

  ➊ row **position** (ROWS windows)
  ➋ row **values** v_i (RANGE windows)

- As the current row changes, the window *slides* with it.

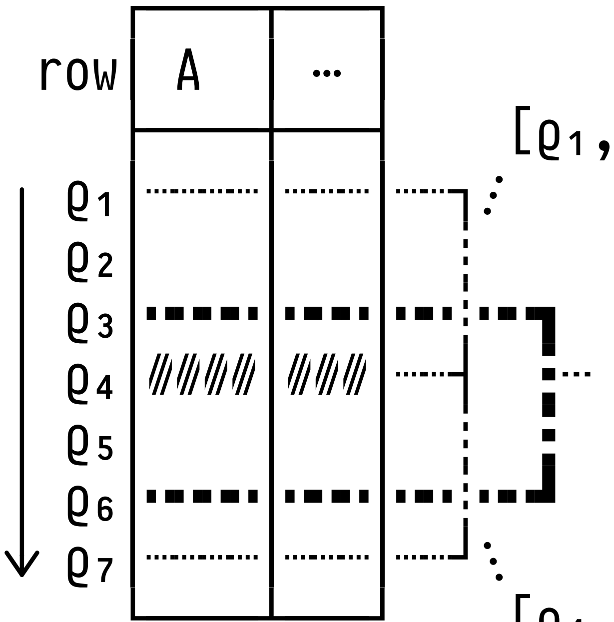
- ⚠️ Window semantics depend on a defined **row ordering**.

Window Frame Specifications (Variant: ROWS)

window function	ordering criteria	frame specification
‹f›	**OVER (ORDER BY** ‹e_1›,…,‹e_n›	[**ROWS** ‹frame›])

‹f› **OVER (ORDER BY** ‹e_1›,…,‹e_n› [**ROWS** ‹frame›])

‹frame›:



[ϱ_1,…,ϱ_4]: **BETWEEN UNBOUNDED PRECEDING AND CURRENT ROW**

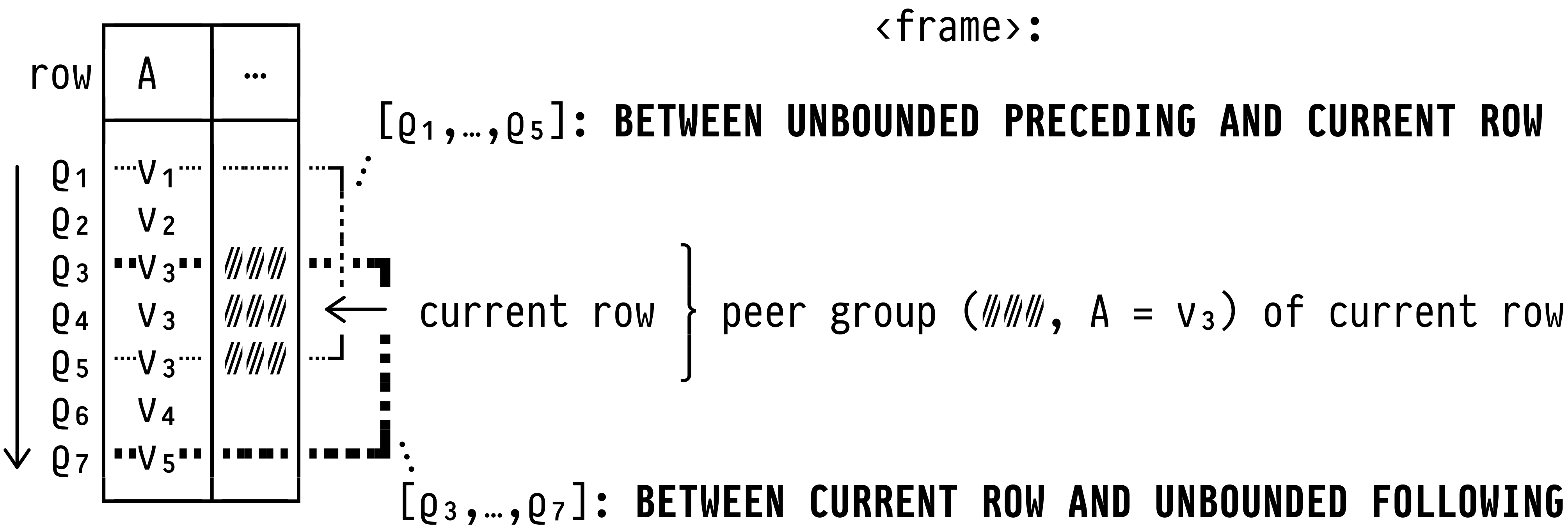
[ϱ_3,…,ϱ_6]: **BETWEEN 1 PRECEDING AND 2 FOLLOWING** (1 = ‹n›, 2 = ‹m›)

[ϱ_4,…,ϱ_7]: **BETWEEN CURRENT ROW AND UNBOUNDED FOLLOWING**

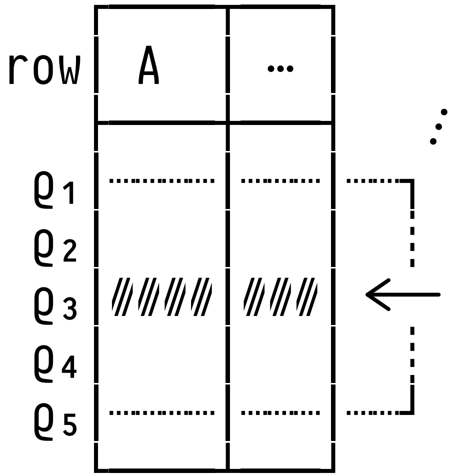
Window Frame Specifications (Variant: RANGE)

window function ... frame specification

<f> OVER (ORDER BY A [**RANGE** <frame>])

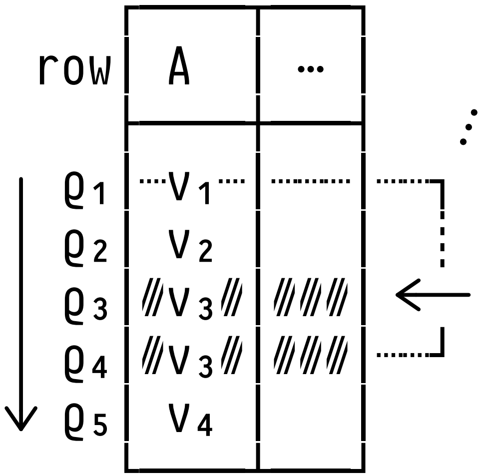


Window Frame Specifications: Abbreviations



$[\varrho_1,\ldots,\varrho_5]$: **OVER ()** ≡ all rows form the window frame (for any current row)

current row



$[\varrho_1,\ldots,\varrho_4]$: **OVER (ORDER BY A)** ≡

```
OVER (ORDER BY A
      RANGE BETWEEN UNBOUNDED PRECEDING
                AND CURRENT ROW)
```

current row

WINDOW Clause: Name the Frame

Syntactic 🍭: If window frame specifications

1. become unwieldy because of verbose SQL syntax and/or
2. one frame is used multiple times in a query,

add a WINDOW clause to a SFW block to **name the frame**, e.g.:

```
SELECT … ‹f› OVER ‹wᵢ› … ‹g› OVER ‹wⱼ› …
FROM   …
WHERE  …
   ⋮
WINDOW ‹w₁› AS (‹frame₁›), …, ‹wₙ› AS (‹frameₙ›)
ORDER BY …
```

Use SQL Itself to Explain Window Frame Semantics ✎

Regular **aggregates** may act as window functions <f>. All **rows in the frame will be aggregated:**

```
SELECT w.row                           AS "current row",
       COUNT(*)             OVER win AS "frame size",
       array_agg(w.row) OVER win AS "rows in frame"
FROM   W AS w
WINDOW win AS (<frame>)
```

Table W

row	a	b
ϱ_1	1	●
ϱ_2	2	○
ϱ_3	3	○
ϱ_4	3	●
⋮	⋮	⋮

2 ¦ PARTITION BY: Window Frames Inside Partitions ✎

Optionally, we may **partition** the input table *before* rows are sorted and window frames are determined:

```
 all input rows that agree on all ‹p_i› form one partition

‹f› OVER ([ PARTITION BY ‹p_1›,…,‹p_m› ]
          [ ORDER BY ‹e_1›,…,‹e_n› ]
          [ ‹frame› ])
```

- Note:

  1. Frames **never cross partitions.**
  2. BETWEEN ⋯ PRECEDING AND ⋯ FOLLOWING **respects partition boundaries.**

Q: What is the Chance of Fine Weather on Weekends? ✎

Input: Daily weather readings in sensors:

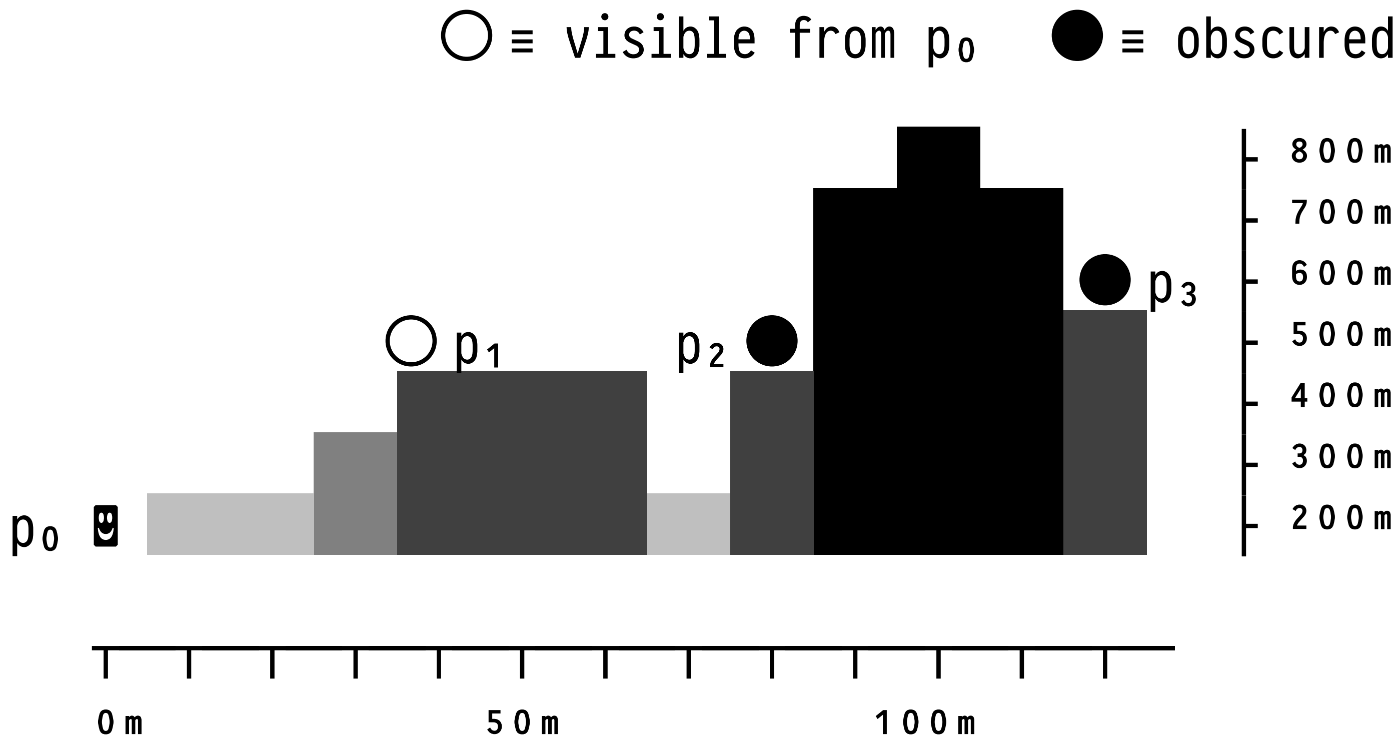
Table sensors

day	weekday	temp	rain
1	Fri	10	800
2	Sat	12	300
⋮	⋮	⋮	⋮

- The weather is fine on day ***d*** if—**on *d* and the two days prior**—the minimum temperature is above 15°C and the overall rainfall is less than 600ml/m².
- **Expected output:**

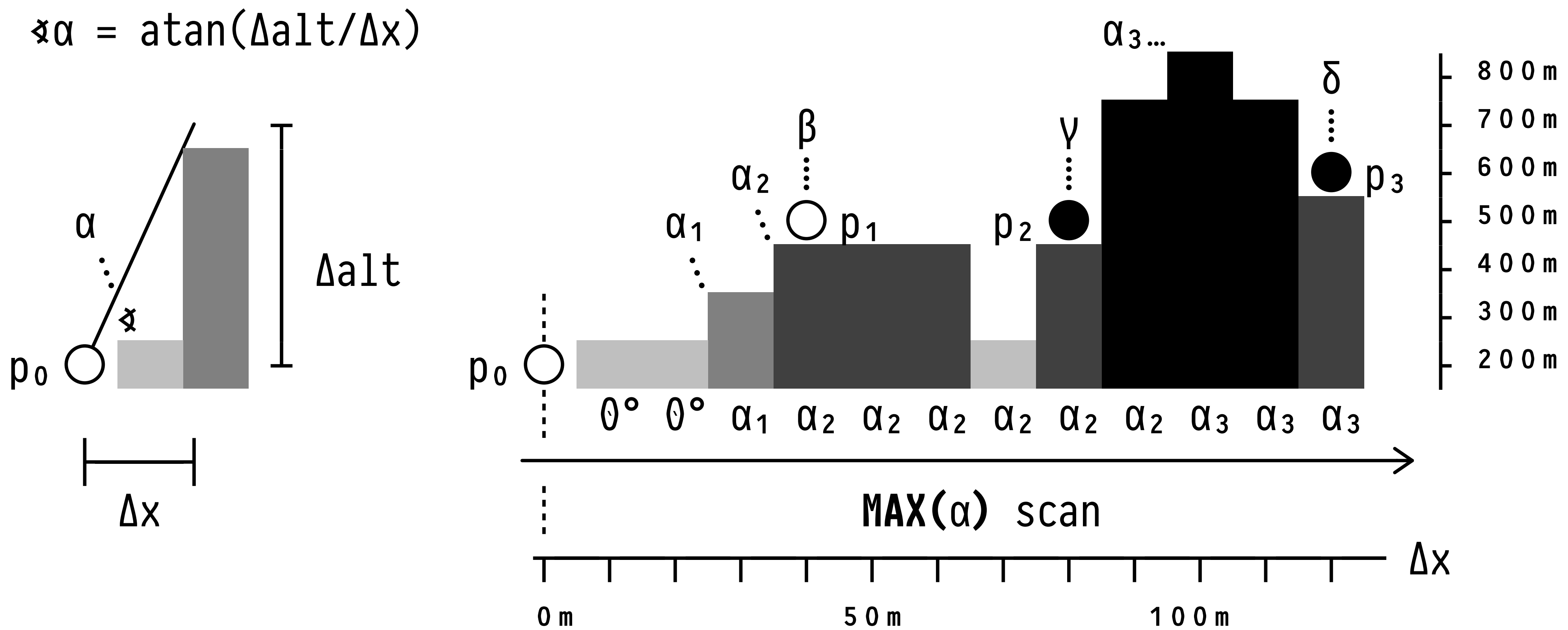
weekend?	% fine
f	29
t	43

🔧 Q: What is Visible in a Hilly Landscape?



- From the viewpoint of p_0 (☻) we can see p_1, but…
  - … p_2 is **obscured** (no straight-line view from p_0),
  - … p_3 is **obscured** (lies behind the 800m peak).

Q: What is Visible in a Hilly Landscape? — A: MAX Scan!



- We have $0° < \alpha_1 < \alpha_2 < \alpha_3$ and $\beta \geqslant \alpha_2$, $\gamma < \alpha_2$, $\delta < \alpha_3$.

  p_1 visible $p_{2,3}$ obscured

Q: What is Visible in a Hilly Landscape? ✎

- **Input:** Location of p_0 (here: x = 0) and 1D-map of hills:

Table map

x	alt
0	200
10	200
⋮	⋮
120	500

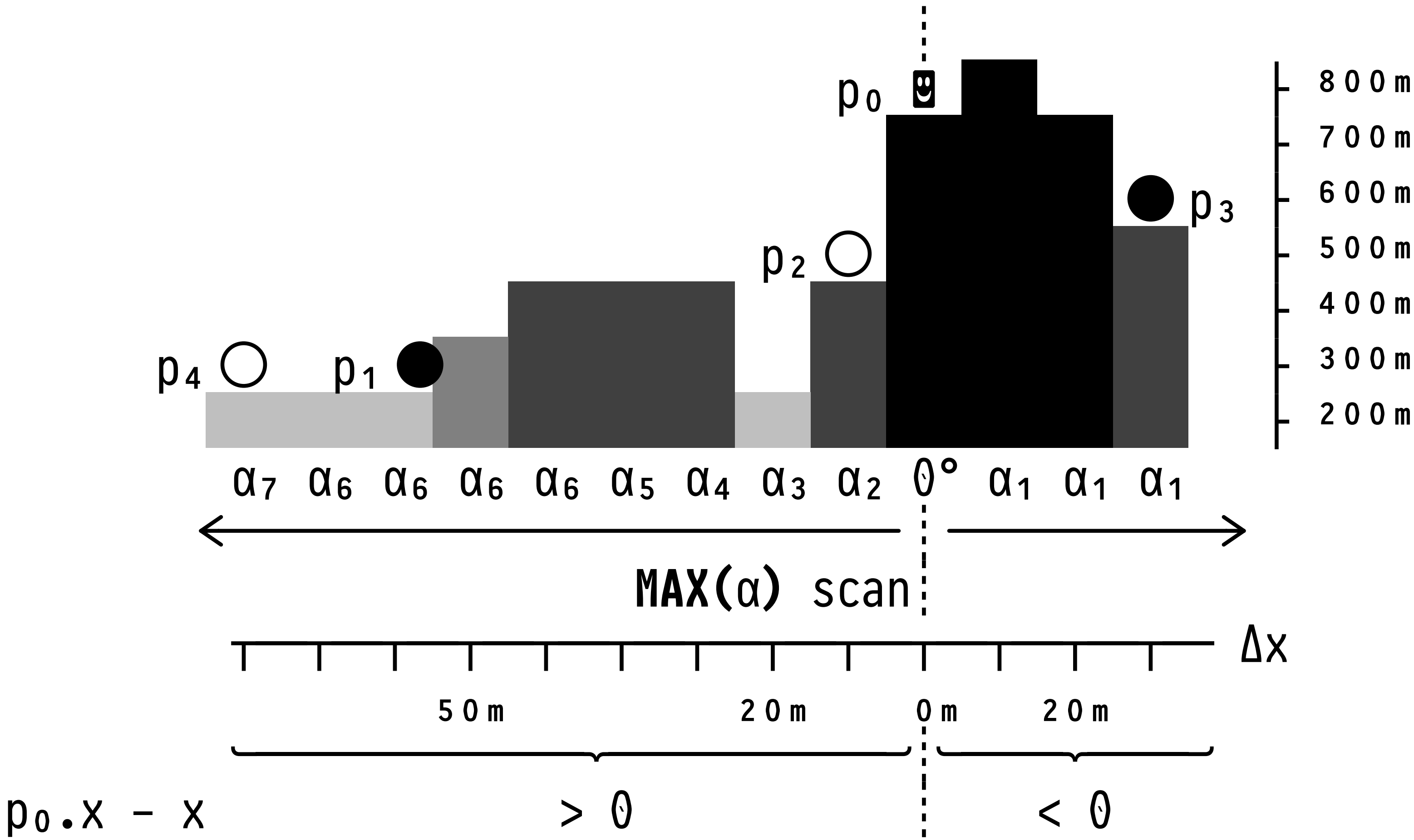
- **Output:** Can p_0 see the point on the hilltop at x?

x	visible?
0	true
10	true
⋮	⋮
120	false

Q: What is Visible in a Hilly Landscape? — MAX Scan ✎

```
WITH
-- ❶ Angles α (in °) between p₀ and the hilltop at x
angles(x, angle) AS (
  SELECT m.x,
         degrees(atan((m.alt - p0.alt) /
                      abs(p0.x - m.x))) AS angle
  FROM   map AS m
  WHERE  m.x > p0.x),
-- ❷ MAX(α) scan (to the right of p₀)
max_scan(x, max_angle) AS (
  SELECT a.x,
         MAX(a.angle)
           OVER (ORDER BY abs(p0.x - a.x)) AS max_angle
  FROM   angles AS a),
⋮
```

Looking Left *and* Right: PARTITION BY



- Need MAX scans left *and* right of p_0 ⇒ use PARTITION BY.

Looking Left *and* Right: PARTITION BY

```
WITH
⋮
-- ❷ MAX(α) scan (left/right of p₀)
max_scan(x, max_angle) AS (
  SELECT a.x,                    --   ∈ {-1, 0 ,1}
         MAX(a.angle)            --  ⏞
           OVER (PARTITION BY sign(p0.x - a.x)
                       ORDER BY abs(p0.x - a.x)) AS max_angle
  FROM   angles AS a  --        ⏟
                      --          Δx > 0
),
⋮
```

- ∀a ∈ angles: a.x ≠ p0.x ⇒ We end up with **two** partitions.

3 ¦ Scans: Not Only in the Hills

Scans are a general and expressive computational pattern:

```
<agg>(<e>) OVER (ORDER BY <e1>, …, <en>
 ︸            {RANGE,ROWS} BETWEEN UNBOUNDED PRECEDING
(ϕ,z,⊕)                              AND CURRENT ROW)
```

- Available in a variety of forms in programming languages
  - Haskell: `scanl z ⊕ xs`, APL: `⊕\xs`, Python: `accumulate`:
    scanl ⊕ z [x_1,x_2,…] = [z, z ⊕ x_1, (z ⊕ x_1) ⊕ x_2, …]

- In parallel programming: *prefix sums* (☞ Guy Blelloch)
  - Sorting, lexical analysis, tree operations, reg.exp. search, drawing operations, image processing, …

Q: Assume *xs* ≡ '((b*2)-4×a×c)*0.5'. What is computed below?

```
SELECT inp.pos, inp.c,
       SUM((array[1,-1,0])[COALESCE(p.oc, 3)])
         OVER (ORDER BY inp.pos) AS d
FROM   unnest(string_to_array(xs, NULL))
         WITH ORDINALITY AS inp(c,pos),
       LATERAL (VALUES (array_position(array['(',')'],
                                       inp.c))) AS p(oc)
ORDER BY inp.pos;
```

💡 **Hint** (this is the same query expressed in APL):

```
xs ← '((b*2)-4×a×c)*0.5'
+\ (1 ¯1 0)['()'⍳xs]
```

```
      window function
           ▼
          <f> OVER ([ PARTITION BY <p₁>,…,<pₘ> ]
                    [ ORDER BY <e₁>,…,<eₙ> ]
                    [ <frame> ])
```

Kinds of **window functions** <f>:

1. **Aggregates:** SUM(•), AVG(•), MAX(•), array_agg(•), … ✓
2. **Row Access:** access row by *absolute/relative position* in ordered frame or partition: first/last/n^{th}/n rows away
3. **Row Ranking:** assign numeric *rank of row* in its partition

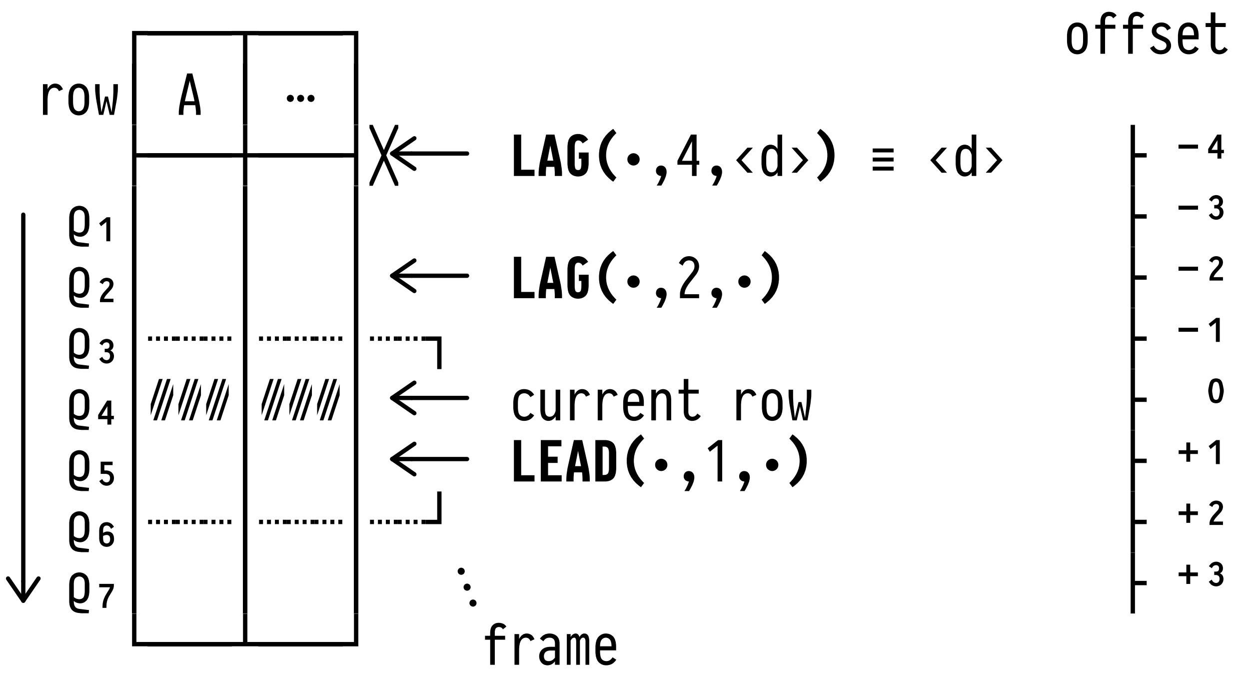
Row access at offset ∓‹n›, relative to the current row:

```
-- evaluate ‹e› as if we were
-- ‹n› rows before the current row
--     ┌──────────┴──────────┐
       LAG(‹e›,‹n›,‹d›) OVER ([ PARTITION BY ‹p₁›,…,‹pₘ› ]
                                 ORDER BY ‹e₁›,…,‹eₙ›
                                 [ ‹frame› ])
```

Note:

- LEAD(‹e›,‹n›,‹d›): ⋯ ‹n› rows **after** the current row ⋯
- Scope is partition—may access rows outside the ‹frame›.
- No row at offset ∓‹n› ⇒ return default ‹d›.

LAG/LEAD: Row Offsets ✎



- The frame of the current row is irrelevant for LAG/LEAD.
- If no default value <d> given ⇒ return NULL.

A March Through the Hills: Ascent or Descent?

```
SELECT m.x, m.alt,
       CASE sign(LEAD(m.alt, 1) OVER rightwards - m.alt)
            WHEN -1 THEN '↘' WHEN 1 THEN '↗'
            WHEN  0 THEN '→' ELSE '?'
       END AS climb,
       LEAD(m.alt, 1) OVER rightwards - m.alt AS "by [m]"
FROM   map AS m
WINDOW rightwards AS (ORDER BY m.x)  --  marching right
```

x	alt	climb	by [m]
0	200	→	0
⋮	⋮	⋮	⋮
90	700	↗	100
100	800	↘	-100
110	700	↘	-200
120	500	?	NULL

Crime Scene: Sessionization

A spy broke into the Police HQ computer system. A log records keyboard activity of user uid at time ts:

Table log

uid	ts
	09-17-2016 07:25:12
	09-17-2016 07:25:18
	09-17-2016 08:01:55
	09-17-2016 08:02:07
	09-17-2016 08:05:30
	09-17-2016 08:05:39
	09-17-2016 08:05:46

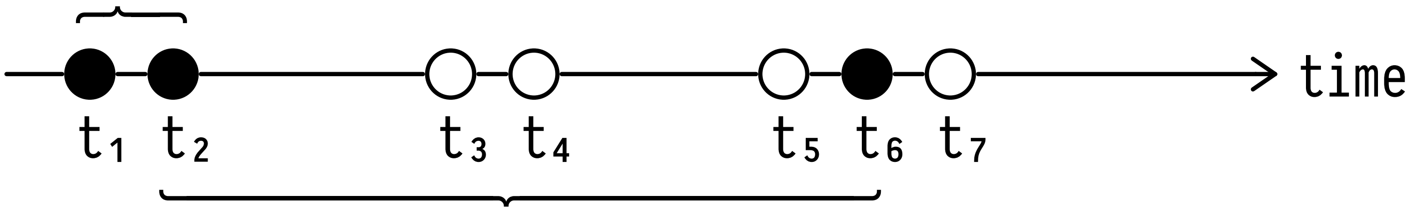
- **Q:** Can we **sessionize** the log so that investigators can identify *sessions* (≡ streaks of uninterrupted activity)?

Sessionization (Query Plan)

1. Cop and spy sessions happen independently (even if concurrent): partition table `log` into 👮 and 🕵 rows.

2. **Tag** keyboard activities (here: 👮):

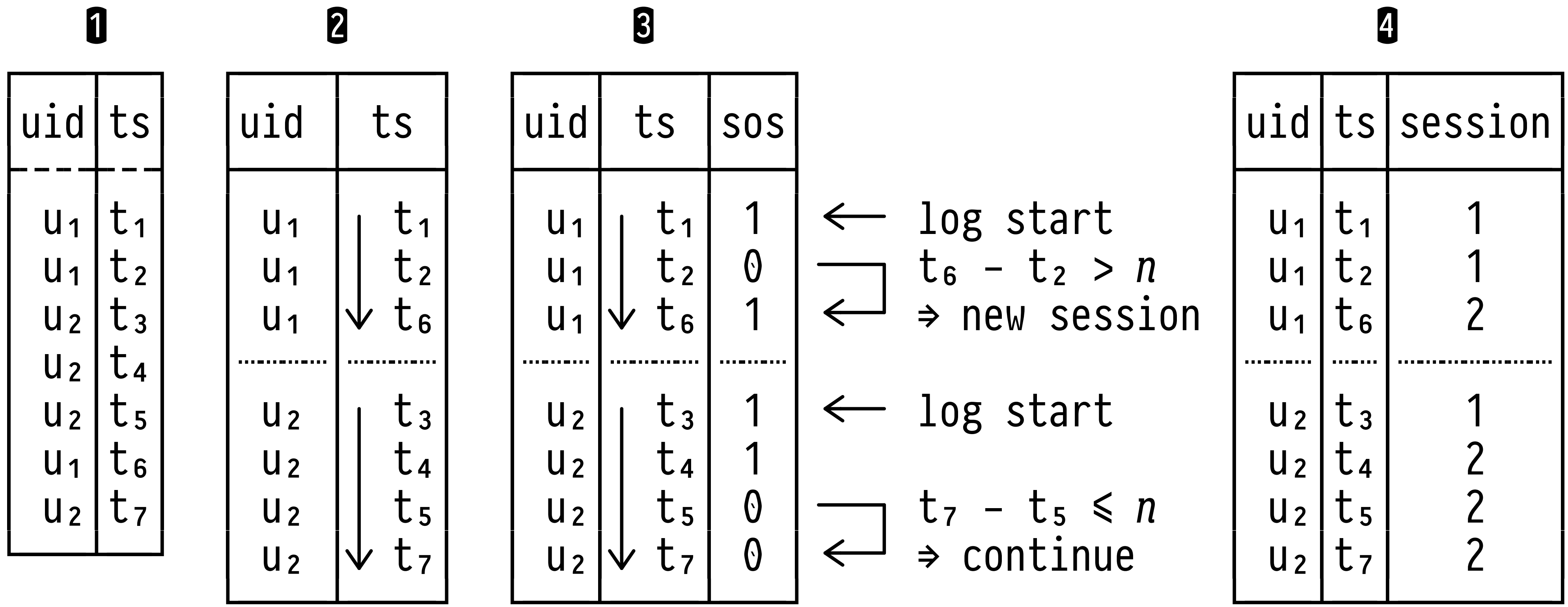
$t_2 - t_1 \leqslant n \Rightarrow$ continue session (tag t_2 with **0**)



$t_6 - t_2 > n \Rightarrow$ new session (tag t_6 with **1**)

3. **Scan** the tagged table and derive session IDs by maintaining a **runnning sum** of *start of session* tags.

Sessionization (Query Plan) ✎



- At log start, always begin a new session.
- How to assign *global session IDs* (u_2's sessions: 3, 4)?

Image Compression by Run-Length Encoding

Compress image by identifying pixel **runs** of the same color:

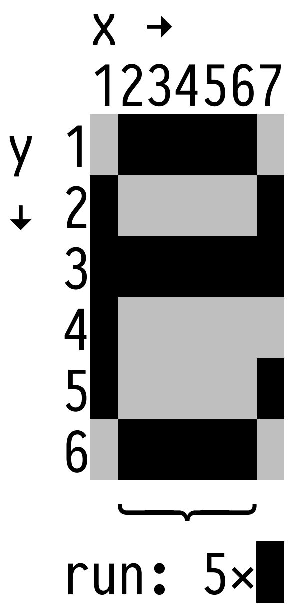


Table **original**

x	y	pixel
1	1	
2	1	
⋮	⋮	⋮
6	6	
7	6	

→

Table **encoding**

run	length	pixel
1	1	
2	5	
⋮	⋮	⋮
12	5	
13	1	

- Here: assumes a row-wise linearization of the pixel map.
- In b/w images we may omit column pixel in table encoding.

Run-Length Encoding (Query Plan) ✎

❶

x	y	pixel	change?	
1	1		t	1
2	1		t	1
3	1		f	0
4	1		f	0
5	1		f	0
6	1		f	0
7	1		t	1
⋮	⋮	⋮	⋮	⋮

❷

x	y	pixel	change?	Σ change?
1	1		1	1
2	1		1	2
3	1		0	2
4	1		0	2
5	1		0	2
6	1		0	2
7	1		1	3
⋮	⋮	⋮	⋮	⋮

run #2 of length 5

❶: LAG(pixel,1,*undefined*): pixel @ (1,1) always “changes.”

❷: SUM() scan of change? may serve as run identifier.

7 ¦ FIRST_VALUE, LAST_VALUE, NTH_VALUE: In-Frame Row Access

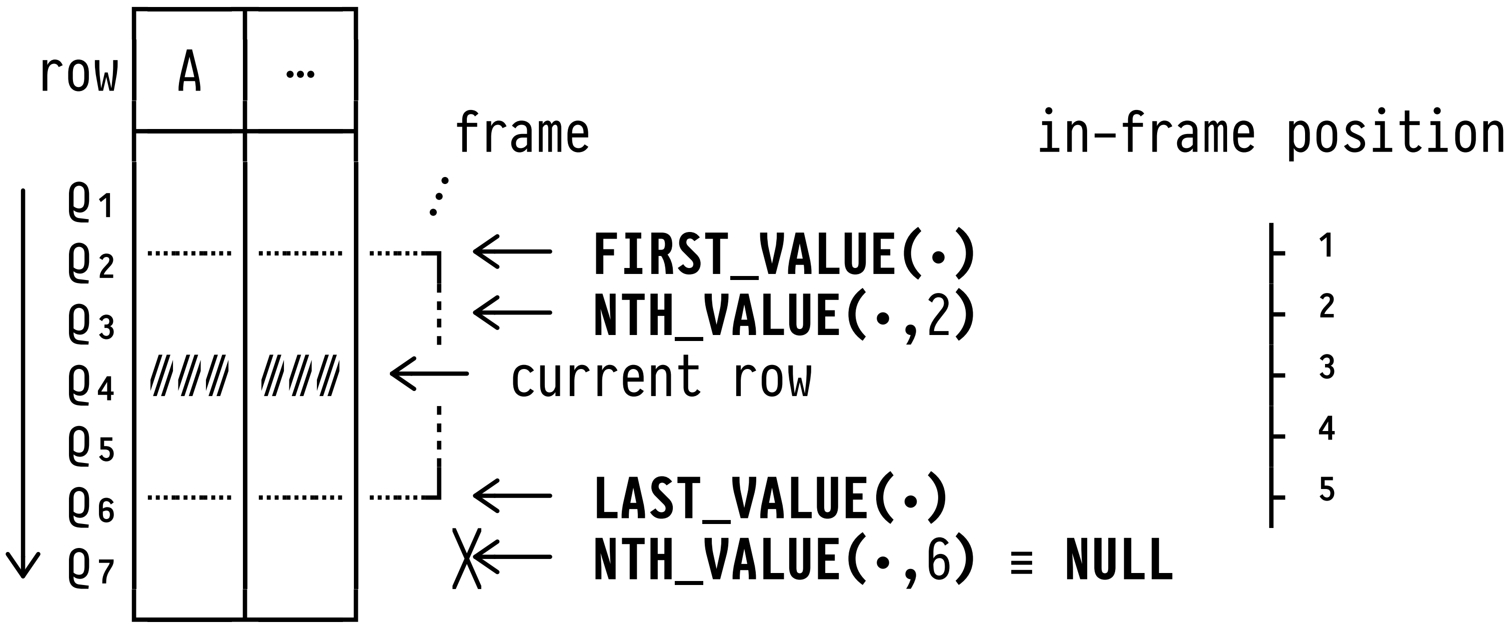
Aggregates reduce *all rows* inside a frame to a single value. Now for something different:

- **Positional access to individual rows** inside a frame is provided by three window functions:

```
-- evaluate expression ‹e› as if we were at
-- the first/last/‹n›th row in the frame
--         ⏞
        FIRST_VALUE(‹e›)       ⎫
        LAST_VALUE(‹e›)        ⎬ OVER (…)
        NTH_VALUE(‹e›,‹n›)     ⎭
```

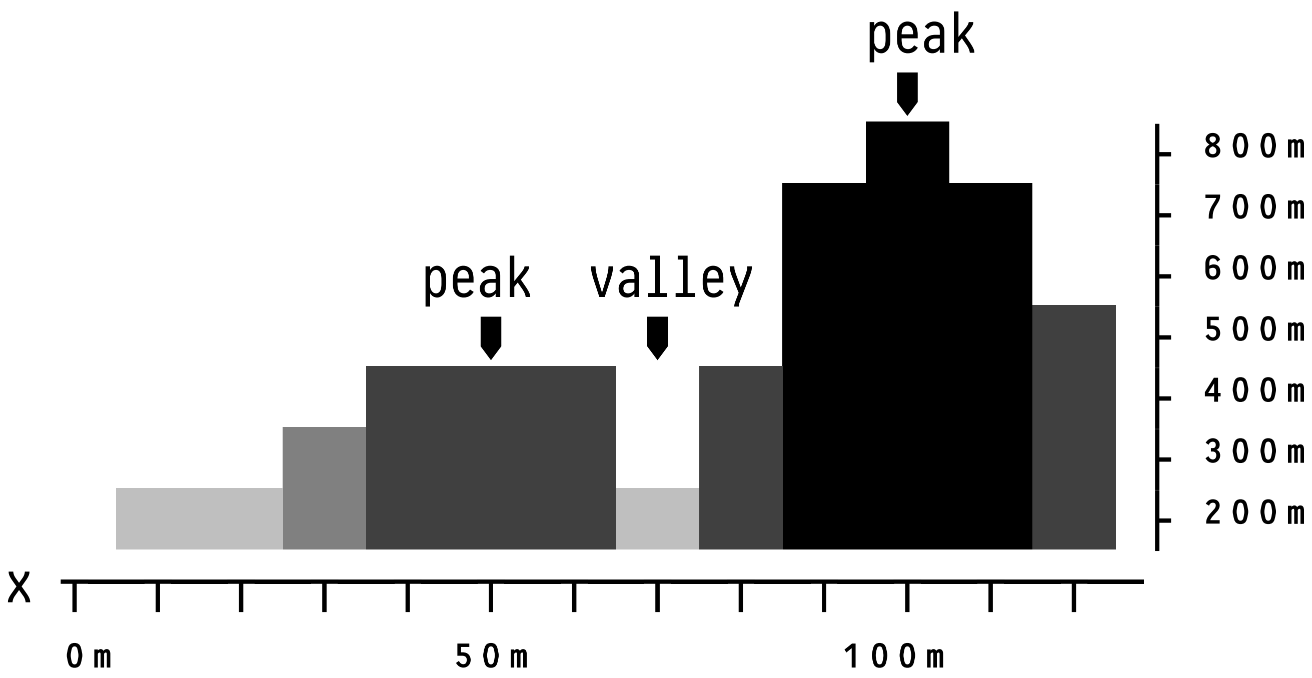
- NTH_VALUE(‹e›,‹n›): No ‹n›th row in frame ⇒ return NULL.

In-Frame Row Access ✎



- FIRST_VALUE(‹e›) ≡ NTH_VALUE(‹e›,1).

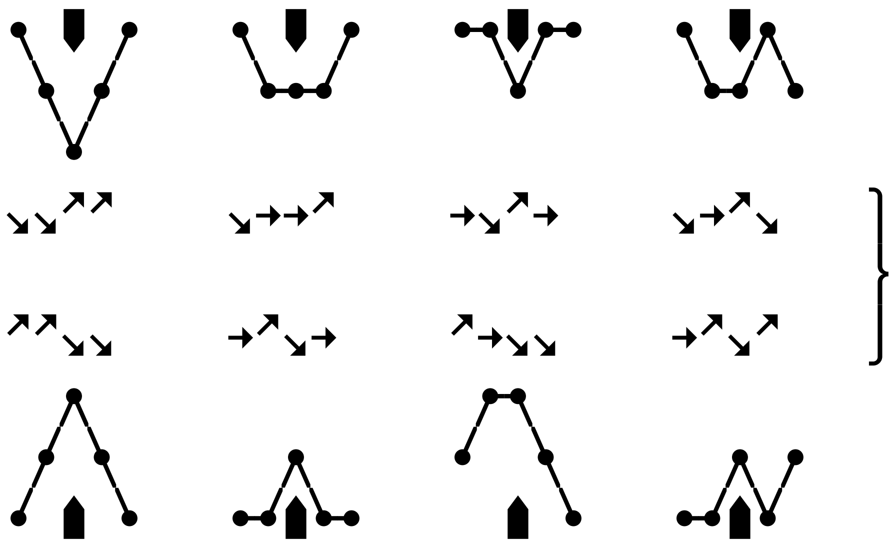
Detecting Landscape Features



- Detect features in hilly landscape. Attach label ∈ {peak,valley,-} to every location x.
- Feature defined by relative altitude change **in vicinity**.

🔧 Detecting Landscape Features (Query Plan)

1. Track relative altitude changes in a sliding **x-window of size 5:**



valley features

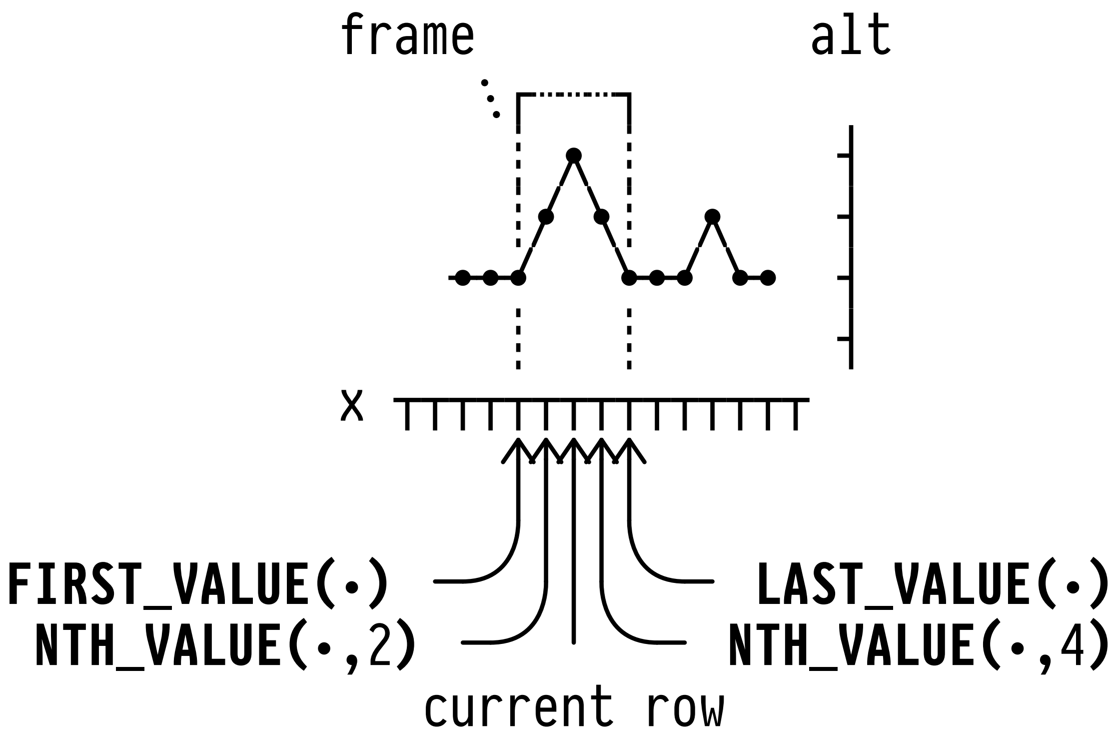
string-encoded
altitude changes
('→↗↘↗' :: text)

peak features

2. **Pattern match** on change strings to detect features.

✎ Altitude Changes in a Sliding Window ✎

- Frame: ROWS BETWEEN 2 PRECEDING AND 2 FOLLOWING (5 rows):



- FIRST_VALUE(alt) < NTH_VALUE(alt,2) ⇒ ascent ('↗').

Altitude Changes in a Sliding Window ✎

```sql
-- Find slopes in -2/+2 vicinity around point x
SELECT m.x, slope(
 sign(FIRST_VALUE(m.alt) OVER w-NTH_VALUE(m.alt,2) OVER w),
 sign(NTH_VALUE(m.alt,2) OVER w-m.alt                    ),
 sign(m.alt                    -NTH_VALUE(m.alt,4) OVER w),
 sign(NTH_VALUE(m.alt,4) OVER w-LAST_VALUE(m.alt)  OVER w)
)
FROM   map AS m
WINDOW w AS (ORDER BY m.x ROWS BETWEEN 2 PRECEDING
                                   AND 2 FOLLOWING)
```

- Recall: 1D landscape represented as table `map(x,alt)`.
- UDF encodes altitude changes: `slope(-1,-1,0,1)` ≡ '↘↘→↗'.

Row Pattern Matching (SQL:2016)

SQL:2016 introduced an entirely new SQL construct, **row pattern matching** (MATCH_RECOGNIZE):

1. ORDER BY: Order the rows of a table.
2. DEFINE: Tag rows that satisfy given predicates.
3. PATTERN: Specify a **regular expression over row tags**, find matches in the ordered sequence of rows.
4. MEASURES: For each match, evaluate expressions that measure its features (matched rows, length, …).

- ⚠️ As of June 2017, not supported by . Implemented in Oracle® 12i only.

Row Pattern Matching (SQL:2016)

```
SELECT *
FROM   map
MATCH_RECOGNIZE (
 ORDER BY x
 MEASURES FIRST(x,1)      AS x,
          MATCH_NUMBER() AS feature,
          CLASSIFIER()   AS slope
 ONE ROW PER MATCH
 AFTER MATCH SKIP TO NEXT ROW
 PATTERN (((DOWN DOWN|DOWN EVEN|UP DOWN|EVEN DOWN)…)
 DEFINE UP   AS UP.alt   > PREV(UP.alt),     --
        DOWN AS DOWN.alt < PREV(DOWN.alt),   --  } row tags
        EVEN AS EVEN.alt = PREV(EVEN.alt)    --
)
```

Output

x	feature	slope
50	1	DOWN
70	2	UP
100	3	DOWN

Countless problem scenarios involve the **number** (position) or **rank** of the current row in an *ordered sequence* of rows.

- Family of **window functions to number/rank rows:**

```
ROW_NUMBER()      ⎫
                  ⎪      --  intra-partition ranking ✓
DENSE_RANK()      ⎪      --  ⏞
RANK()            ⎬ OVER ([ PARTITION BY <p₁>,…,<pₘ> ]
                  ⎪         [ ORDER BY <e₁>,…,<eₙ> ] )
PERCENT_RANK()    ⎪      --  ⏟
CUME_DIST()       ⎪      -- ranking w/o ORDER BY ϟ
NTILE(<n>)        ⎭
```

- Scope is partition (if present) — <frame> is irrelevant.

Numbering and Ranking Rows — ‹f› OVER (ORDER BY A) ✎

Table W

row	A	**ROW_NUMBER**	**DENSE_RANK**	**RANK**
ϱ1	1	1	1	1
ϱ2	2	2	2	2
ϱ3	**3**	3	3	3
ϱ4	**3**	4	3	3
ϱ5	**3**	5	3	3
ϱ6	4	6	4	6
ϱ7	**5**	7	5	7
ϱ8	**5**	8	5	7
ϱ9	6	9	6	9

┈ rows that agree on
┈ the sort criterion
┈ (here: **A**) rank equally

I mind the ranking gap
(think Olympics)

- DENSE_RANK() ⩽ RANK() ⩽ ROW_NUMBER()

Once More: Find the Top *n* Rows in a Group

Table dinosaurs

species	length	height	legs
⋮	⋮	⋮	∈ {2,4,NULL}

```
SELECT tallest.legs, tallest.species, tallest.height
FROM (SELECT d.legs, d.species, d.height,
ROW_NUMBER() RANK() OVER (PARTITION BY d.legs
                               ORDER BY d.height DESC) AS n
      FROM  dinosaurs AS d
      WHERE d.legs IS NOT NULL) AS tallest
WHERE n <= 3
```

- RANK() *vs* ROW_NUMBER(): both OK, but different semantics!
- Need a subquery: window functions *not* allowed in WHERE.

Identify Consecutive Ranges

- What you often encounter in scientific papers:
  “… *as Knuth has shown in [5,2,14,3,1,42,6,10,7,13]* …”

- What you want to see:
  “… *as Knuth has shown in [1–3,5–7,10,13&14,42]* …”

Table **citations**

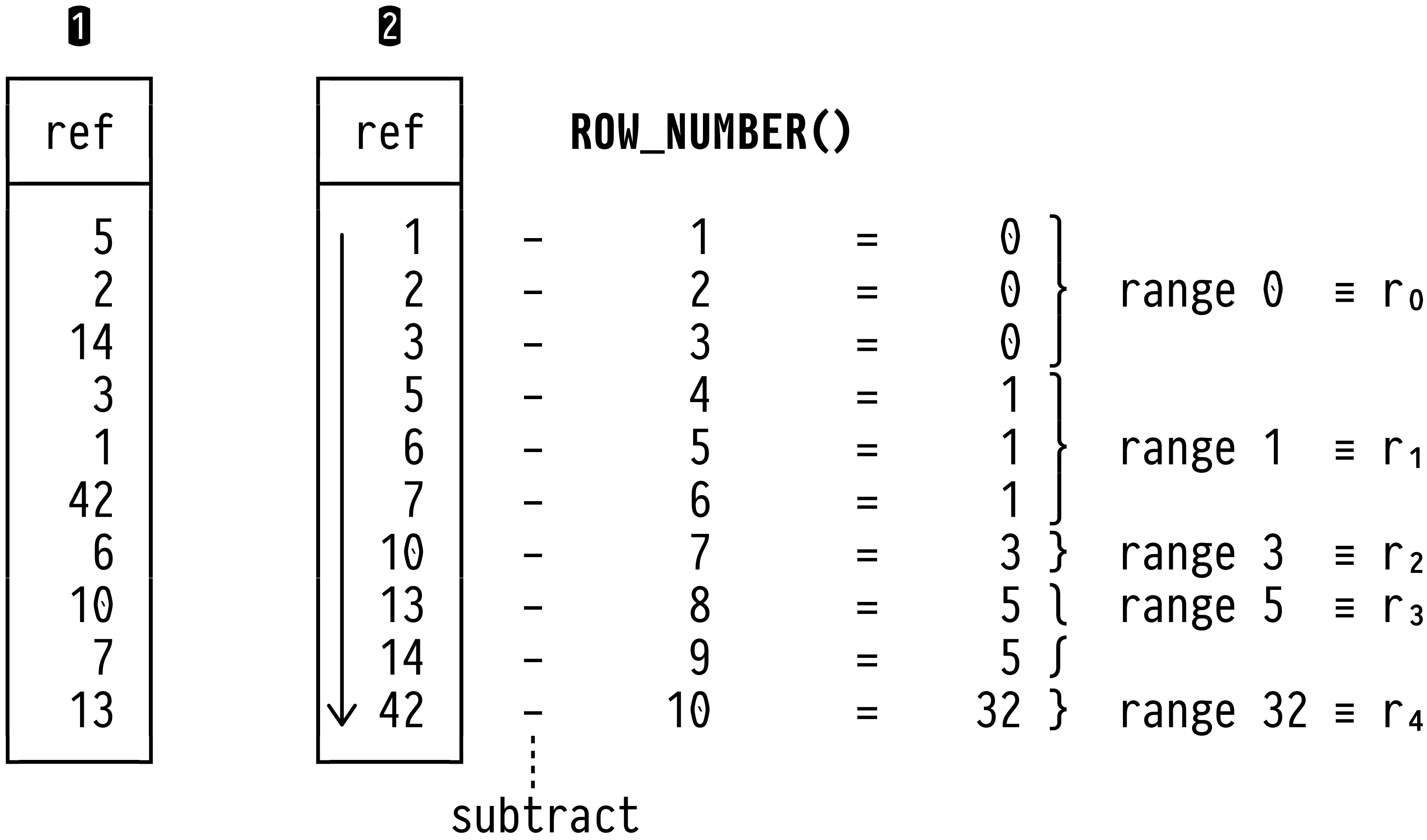
ref
5
2
⋮
13

Output

ref	range
1	r_0
2	r_0
⋮	⋮
42	r_4

references belong to the same range

Identify Consecutive Ranges (Query Plan) ✎



Numbering and Ranking Rows — ‹f› OVER (ORDER BY A) ✎

row	A	PERCENT_RANK	CUME_DIST	NTILE(3)
ϱ1	1	0	1/9	1
ϱ2	2	1/8	2/9	1
ϱ3	**3**	2/8	5/9	1
ϱ4	**3**	2/8	5/9	2
ϱ5	**3**	2/8	5/9	2
ϱ6	4	5/8	6/9	2
ϱ7	**5**	6/8	8/9	3
ϱ8	**5**	6/8	8/9	3
ϱ9	6	8/8	9/9	3

⋯ rows that agree on
⋯ the sort criterion
⋯ (here: **A**) rank equally

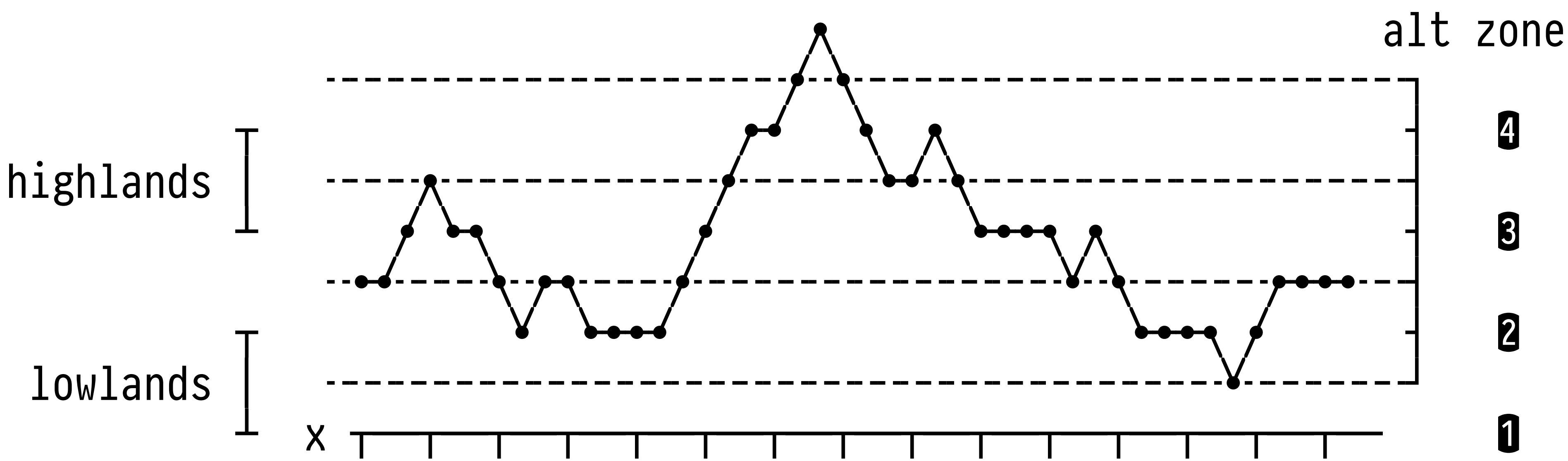
current row is in the *n*th
of 3 chunks of rows

n% of the other rows rank
lower than the current row

the current row and lower ranked
rows make up *n*% of all rows

Altitudinal Mountain Zones

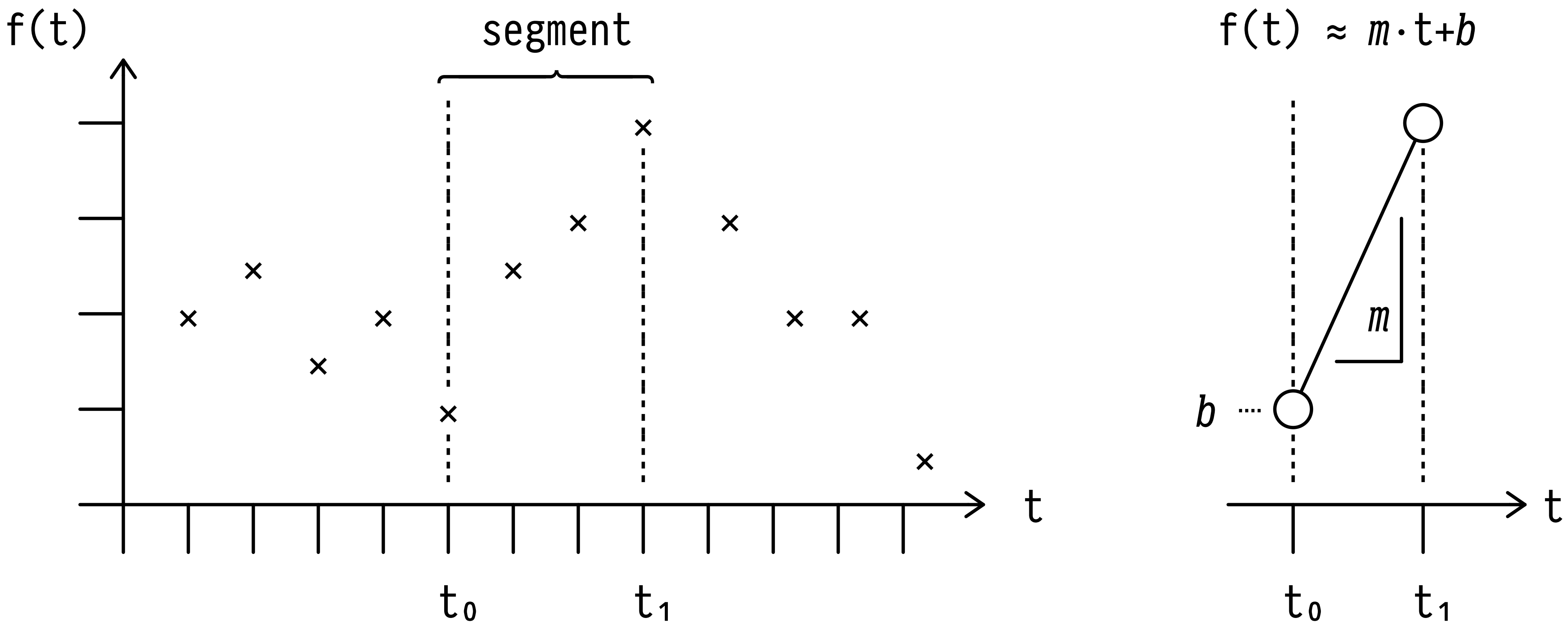
- Classify the altitudes of a mountain range into
  1. **equal-sized** vegetation **zones** and
  2. lowlands (altitude in the lowest **20%**) and highlands (between **60%–80%** of maximum altitude).



🔧 Altitudinal Mountain Zones ✎

```sql
-- Classify altitudinal zones in table mountains(x, alt)
--
SELECT
  m.x, m.alt,
  NTILE(4) OVER altitude AS zone,
  CASE
    WHEN PERCENT_RANK() OVER altitude BETWEEN 0.6 AND 0.8
         THEN 'highlands'
    WHEN PERCENT_RANK() OVER altitude < 0.2
         THEN 'lowlands'
         ELSE '-'
  END AS region
FROM   mountains AS m
WINDOW altitude AS (ORDER BY m.alt)
ORDER BY m.x;
```

Linear Approximation of a Time Series ✎



1. NTILE(‹n›) **segments** time series at desired granularity.
2. Compute m, b in each **segment ≡ window frame**.

Summary: Window Function Semantics[1]

Scope	Computation	Function	Description
frame	aggregation	(aggregates)	SUM, AVG, MAX, array_agg, …
⋮	row access	FIRST_VALUE(*e*)	*e* at first row in frame
⋮	⋮	LAST_VALUE(*e*)	*e* at last row in frame
⋮	⋮	NTH_VALUE(*e*,*n*)	*e* at n^{th} row in frame
partition	row access	LAG(*e*,*n*,*d*)	*e* at *n* rows *before* current row
⋮	⋮	LEAD(*e*,*n*,*d*)	*e* at *n* rows *after* current row
⋮	ranking	ROW_NUMBER()	number of current row
⋮	⋮	RANK()	rank with gaps ("Olympics")
⋮	⋮	DENSE_RANK()	rank without gaps
⋮	⋮	PERCENT_RANK()	relative rank of current row
⋮	⋮	CUME_DIST()	ratio of rows up to –"–
⋮	⋮	NTILE(*n*)	rank on a scale {1,2,…,*n*}

[1] FIRST_VALUE(*e*): expression *e* will be evaluated *as if* we are at the first row in the frame.
LAG(*e*,*n*,*d*): default expression *d* is returned if there is no row at offset *n* before the current row.