Chapter 7

External Sorting

Sorting Tables Larger Than Main Memory

Architecture and Implementation of Database Systems

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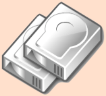
Torsten Grust
Wilhelm-Schickard-Institut für Informatik
Universität Tübingen



Query Processing

Challenges lurking behind a SQL query

```
SELECT  C.CUST_ID, C.NAME, SUM(O.TOTAL) AS REVENUE
  FROM  CUSTOMERS AS C, ORDERS AS O
 WHERE  C.ZIPCODE BETWEEN 8000 AND 8999
   AND  C.CUST_ID = O.CUST_ID
 GROUP  BY C.CUST_ID
 ORDER  BY C.CUST_ID, C.NAME
```

aggregation — SUM(O.TOTAL)

selection — C.ZIPCODE BETWEEN 8000 AND 8999

grouping — GROUP BY C.CUST_ID

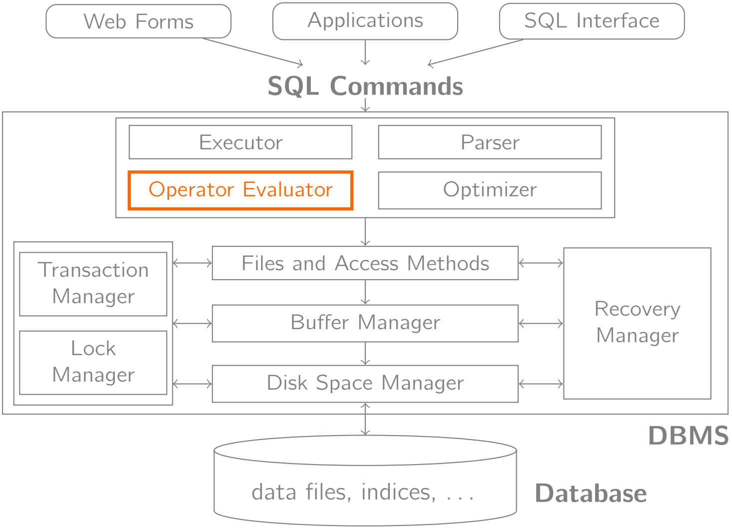
join — C.CUST_ID = O.CUST_ID

sorting — ORDER BY C.CUST_ID, C.NAME

A DBMS **query processor** needs to perform a number of tasks

- with **limited memory resources**,
- over **large amounts of data**,
- yet **as fast as possible**.

Query Processing



Query Plans and Operators

Query plans and operators

- DBMS does *not* execute a query as a large monolithic block but rather provides a number of specialized routines, the **query operators**.
- Operators are "plugged together" to form a network of operators, a **plan**, that is capable of evaluating a given query.
- Each operator is carefully implemented to perform a specific task well (*i.e.*, time- and space-efficient).

- **Now:** Zoom in on the details of the implementation of one of the most basic and important operators: **sort**.

Query Processing: Sorting

- **Sorting** stands out as a useful operation, explicit or implicit:

Explicit sorting via the SQL `ORDER BY` clause

```
SELECT   A,B,C
FROM     R
ORDER BY A
```

Implicit sorting, e.g., for duplicate elimination

```
SELECT DISTINCT A,B,C
FROM            R
```

Implicit sorting, e.g., to prepare equi-join

```
SELECT R.A,S.Y
FROM   R,S
WHERE  R.B = S.X
```

- Further:
  Grouping via `GROUP BY`, B^+-tree bulk loading, sorted *rid* scans after access to unclustered indexes, . . .

Sorting

Sorting

- A file is **sorted** with respect to **sort key** k and **ordering** θ, if for any two records $r_{1,2}$ with r_1 preceding r_2 in the file, we have that their correspoding keys are in θ-order:

$$r_1 \,\theta\, r_2 \quad\Leftrightarrow\quad r_1.k \,\theta\, r_2.k \enspace .$$

- A key may be a single attribute as well as an ordered list of attributes. In the latter case, order is defined **lexciographically**. Consider: $k = (\mathtt{A}, \mathtt{B})$, $\theta = <$:

$$r_1 < r_2 \quad\Leftrightarrow\quad \begin{array}{l} r_1.\mathtt{A} < r_2.\mathtt{A} \lor \\ (r_1.\mathtt{A} = r_2.\mathtt{A} \land r_1.\mathtt{B} < r_2.\mathtt{B}) \enspace . \end{array}$$

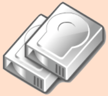
Sorting

- As it is a principal goal not to restrict the file sizes a DBMS can handle, we face a fundamental problem:

  How can we sort a file of records whose **size exceeds the available main memory space** *(let alone the available buffer manager space) by far?*

- Approach the task in a two-phase fashion:
  1. Sorting a file of arbitrary size is possible even if **three pages** of buffer space is all that is available.
  2. Refine this algorithm to make effective use of larger and thus more realistic buffer sizes.

- As we go along, consider a number of further optimizations in order to **reduce the overall number of required page I/O operations**.



Two-Way Merge Sort

We start with **two-way merge sort**, which can sort files of arbitrary size with only **three pages** of buffer space.

Two-way merge sort

Two-way merge sort sorts a file with $N = 2^k$ pages in multiple **passes**, each of them producing a certain number of sorted sub-files called **runs**.

- **Pass 0** sorts each of the 2^k input pages individually and in **main memory**, resulting in 2^k sorted runs.
- **Subsequent passes merge** pairs of runs into larger runs. Pass n produces 2^{k-n} runs.
- **Pass** k leaves only one run: the sorted overall result.

During each pass, we consult every page in the file. Hence, $k \cdot N$ page reads and $k \cdot N$ page writes are required to sort the file.

Basic Two-Way Merge Sort Idea

Pass 0 (**Input:** $N = 2^k$ unsorted pages; **Output:** 2^k sorted runs)

1. **Read** N pages, **one page at a time**
2. **Sort** records, page-wise, in main memory.
3. **Write** sorted pages to disk (each page results in a **run**).

This pass requires **one page** of buffer space.

Pass 1 (**Input:** $N = 2^k$ sorted runs; **Output:** 2^{k-1} sorted runs)

1. Open two runs r_1 and r_2 from Pass 0 for reading.
2. **Merge** records from r_1 and r_2, reading input page-by-page.
3. **Write** new two-page run to disk (page-by-page).

This pass requires **three pages** of buffer space.

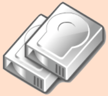
⋮

Pass n (**Input:** 2^{k-n+1} sorted runs; **Output:** 2^{k-n} sorted runs)

1. Open two runs r_1 and r_2 from Pass $n-1$ for reading.
2. **Merge** records from r_1 and r_2, reading input page-by-page.
3. **Write** new 2^n-page run to disk (page-by-page).

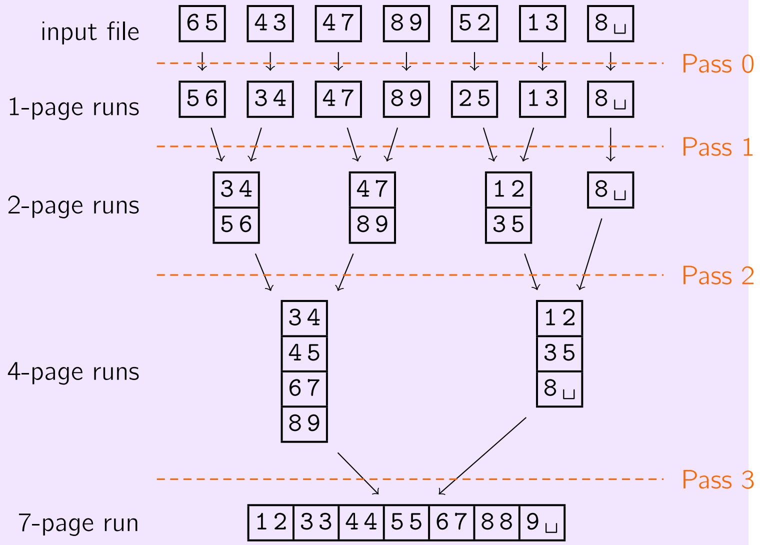
This pass requires **three pages** of buffer space.

⋮



Two-way Merge Sort: Example

Example (7-page file, two records per page, keys k shown, $\theta = <$)

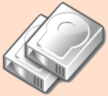


Two-Way Merge Sort: Algorithm

Two-way merge sort, $N = 2^k$

```
Function: two_way_merge_sort (file, N)
/* Pass 0: create N sorted single-page runs
   (in-memory sort)                                  */
foreach page p in file do
    read p into memory, sort it, write it out into a new run;
/* next k passes merge pairs of runs, until only one
   run is left                                       */
for n in 1 ... k do
    for r in 0 ... 2^(k−n) − 1 do
        merge runs 2 · r and 2 · r + 1 from previous pass into a
        new run, reading the input runs one page at a time;
        delete input runs 2 · r and 2 · r + 1 ;
result ← last output run;
```

Each merge requires **three buffer frames** (two to read the two input files and one to construct output pages).



Two-Way Merge Sort: I/O Behavior

- To sort a file of N pages, in each pass we read N pages, sort/merge, and write N pages out again:

$$2 \cdot N \quad \text{I/O operations per pass}$$

- Number of passes:

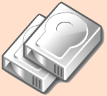
$$\underbrace{1}_{\text{Pass } 0} + \underbrace{\lceil \log_2 N \rceil}_{\text{Passes } 1, \ldots, k}$$

- **Total number of I/O operations:**

$$2 \cdot N \cdot (1 + \lceil \log_2 N \rceil)$$

✎ **How many I/Os does it take to sort an 8 GB file?**

Assume a page size of 8 KB (with 1000 records each).



External Merge Sort

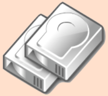
- So far we have "voluntarily" used only three pages of buffer space.

  How could we **make effective use of a significantly larger buffer page pool** *(of, say, B frames)?*

- Basically, there are two knobs we can turn and tune:

  1. **Reduce the number of initial runs** by using the full buffer space during the in-memory sort.

  2. **Reduce the number of passes** by merging more than two runs at a time.



Reducing the Number of Initial Runs

With B frames available in the buffer pool, **we can read B pages at a time during Pass 0** and sort them in memory (↗ slide 9):

Pass 0 (**Input:** N unsorted pages; **Output:** $\lceil N/B \rceil$ sorted runs)

1. **Read** N pages, B **pages at a time**
2. **Sort** records in main memory.
3. **Write** sorted pages to disk (resulting in $\lceil N/B \rceil$ **runs**).

This pass uses B **pages** of buffer space.

The **number of initial runs** determines the **number of passes** we need to make (↗ slide 12):

⇒ **Total number of I/O operations:**

$$2 \cdot N \cdot (1 + \lceil \log_2 \lceil N/B \rceil \rceil) \quad .$$

✎ **How many I/Os does it take to sort an 8 GB file now?**

Again, assume 8 KB pages. Available buffer space is $B = 1{,}000$.

Reducing the Number of Passes

With B frames available in the buffer pool, we can **merge** $B - 1$ **pages at a time** (leaving one frame as a write buffer).

Pass n (**Input:** $\frac{\lceil N/B \rceil}{(B-1)^{n-1}}$ sorted runs; **Output:** $\frac{\lceil N/B \rceil}{(B-1)^{n}}$ sorted runs)

1. Open $B - 1$ runs $r_1 \dots r_{B-1}$ from Pass $n - 1$ for reading.
2. **Merge** records from $r_1 \dots r_{B-1}$, reading page-by-page.
3. **Write** new $B \cdot (B-1)^n$-page run to disk (page-by-page).

This pass requires B **pages** of buffer space.

With B pages of buffer space, we can do a $(B - 1)$-**way merge**.

⇒ **Total number of I/O operations:**

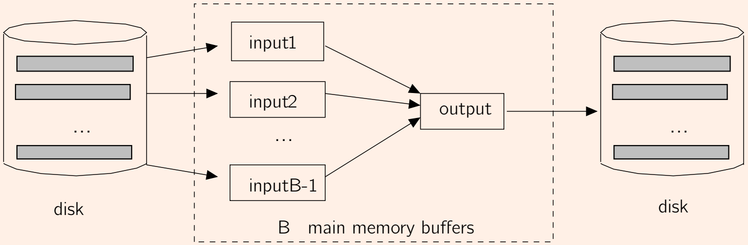
$$2 \cdot N \cdot \left(1 + \lceil \log_{B-1} \lceil N/B \rceil \rceil\right) \enspace .$$

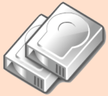
✎ **How many I/Os does it take to sort an 8 GB file now?**

Again, assume 8 KB pages. Available buffer space is $B = 1{,}000$.

Reducing the Number of Passes

$(B - 1)$-way merge using a buffer of B pages

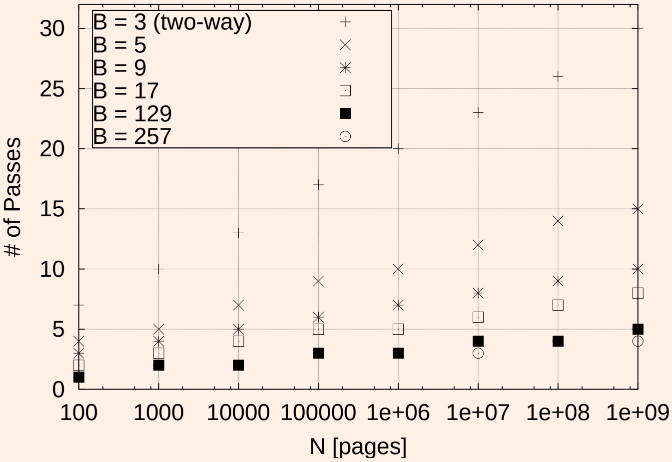




External Sorting: I/O Behavior

- The I/O savings in comparison to two-way merge sort ($B = 3$) can be substantial:

of passes for buffers of size $B = 3, 5, \ldots, 257$



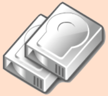
External Sorting: I/O Behavior

- Sorting N pages with B buffer frames requires

$$2 \cdot N \cdot \left(1 + \lceil \log_{B-1} \lceil {}^{N}/_{B} \rceil \rceil \right)$$

I/O operations.

✎ **What is the access pattern of these I/Os?**



Blocked I/O

We could improve the I/O pattern by reading **blocks** of, say, b pages at once during the **merge** phases.

- Allocate b pages for each input (instead of just one).
- **Reduces per-page I/O cost** by a factor of $\approx b$.
- The price we pay is a **decreased fan-in** (resulting in an increased number of passes and more I/O operations).
- In practice, main memory sizes are typically large enough to sort files with **just one merge pass**, even with blocked I/O.

✎ How long does it take to sort 8 GB (counting only I/O cost)?

Assume 1,000 buffer pages of 8 KB each, 8.5 ms average seek time.

- Without blocked I/O: $\approx 4 \cdot 10^6$ disk seeks (9.9 h) + transfer of $\approx 6 \cdot 10^6$ disk pages (14.1 min)
- With blocked I/O ($b = 32$ page blocks): $\approx 6 \cdot 32,800$ disk seeks (28.1 min) + transfer of $\approx 8 \cdot 10^6$ disk pages (18.8 min)

External Merge Sort: CPU Load and Comparisons

- External merge sort reduces the I/O load, but is considerably **CPU intensive**.
- Consider the $(B-1)$-**way merge** during passes $1, 2, \ldots$: To pick the next record to be moved to the output buffer, we need to perform $B - 2$ comparisons.

Example (Comparisons for $B - 1 = 4, \theta = <$)

$$\left\{\begin{array}{l} 087\ 503\ 504\ \ldots \\ 170\ 908\ 994\ \ldots \\ 154\ 426\ 653\ \ldots \\ 612\ 613\ 700\ \ldots \end{array}\right. \rightsquigarrow 087 \quad \left\{\begin{array}{l} 503\ 504\ \ldots \\ 170\ 908\ 994\ \ldots \\ 154\ 426\ 653\ \ldots \\ 612\ 613\ 700\ \ldots \end{array}\right. \rightsquigarrow 087\ 154 \quad \left\{\begin{array}{l} 503\ 504\ \ldots \\ 170\ 908\ 994\ \ldots \\ 426\ 653\ \ldots \\ 612\ 613\ 700\ \ldots \end{array}\right. \rightsquigarrow$$

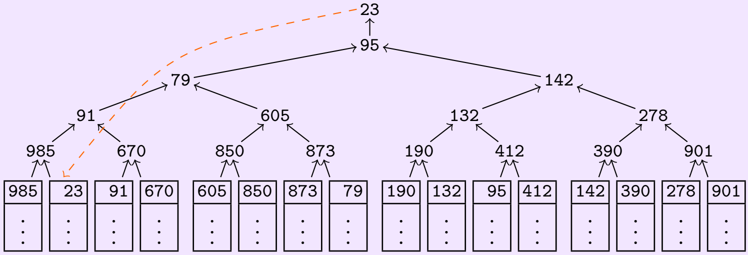
$$087\ 154\ 170 \quad \left\{\begin{array}{l} 503\ 504\ \ldots \\ 908\ 994\ \ldots \\ 426\ 653\ \ldots \\ 612\ 613\ 700\ \ldots \end{array}\right. \rightsquigarrow 087\ 154\ 170\ 426 \quad \left\{\begin{array}{l} 503\ 504\ \ldots \\ 908\ 994\ \ldots \\ 653\ \ldots \\ 612\ 613\ 700\ \ldots \end{array}\right. \quad \ldots$$

Selection Trees

Choosing the next record from $B-1$ (or $B/b-1$) input runs can be quite CPU intensive ($B-2$ comparisons).

- Use a **selection tree** to reduce this cost.
- *E.g.*, "tree of losers" (↗ D. Knuth, TAoCP, vol. 3):

Example (Selection tree, read bottom-up)



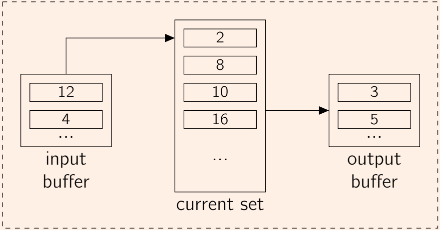
- This cuts the number of comparisons down to $\log_2(B-1)$.

Further Reducing the Number of Initial Runs

- **Replacement sort** can help to further cut down the number of initial runs $\lceil N/B \rceil$: try to **produce initial runs with more than B pages**.

Replacement sort

- Assume a buffer of B pages. Two pages are dedicated **input** and **output buffers**. The remaining $B - 2$ pages are called the **current set**:



Replacement Sort

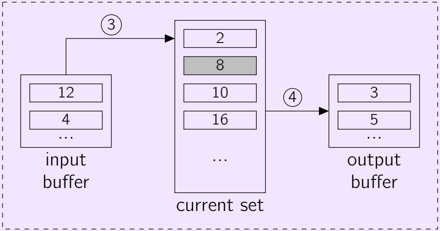
Replacement sort

1. Open an empty run file for writing.
2. Load next page of file to be sorted into input buffer.
   If input file is exhausted, go to 4.
3. While there is space in the current set, move a record from input buffer to current set (if the input buffer is empty, reload it at 2).
4. In current set, pick record r with smallest key value k such that $k \geqslant k_{out}$ where k_{out} is the maximum key value in output buffer.[1] Move r to output buffer. If output buffer is full, append it to current run.
5. If all k in current set are $< k_{out}$, append output buffer to current run, close current run. Open new empty run file for writing.
6. If input file is exhausted, stop. Otherwise go to 3.

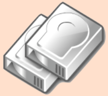
[1] If output buffer is empty, define $k_{out} = -\infty$.

Replacement Sort

Example (Record with key $k = 8$ will be the next to be moved into the output buffer; current $k_{out} = 5$)



- The record with key $k = 2$ remains in the current set and will be written to the subsequent run.



Replacement Sort

✎ Tracing replacement sort

Assume $B = 6$, *i.e.*, a current set size of 4. The input file contains records with `INTEGER` key values:

503 087 512 061 908 170 897 275 426 154 509 612 .

Write a trace of replacement sort by filling out the table below, mark the end of the current run by ⟨`EOR`⟩ (the current set has already been populated at step ③):

current set				output
503	087	512	061	

Replacement Sort

- Step ④ of replacement sort will benefit from techniques like selection tree, esp. if $B - 2$ is large.
- The replacement sort trace suggests that the length of the initial runs indeed increases. In the example: first run length 7 $\approx$ **twice the size of the current set**.

✎ Length of initial runs?

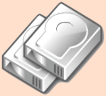
Implement replacement sort to empricially determine initial run length or check the proper analysis (↗ D. Knuth, TAoCP, vol. 3, p. 254).

External Sort: Remarks

- External sorting follows a **divide and conquer** principle.
  - This results in a number of **indepdendent (sub-)tasks.**
  - **Execute tasks in parallel** in a distributed DBMS or exploit multi-core parallelism on modern CPUs.

- To keep the CPU busy while the input buffer is reloaded (or the output buffer appended to the current run), use **double buffering**:

  Create **shadow buffers** for the input and output buffers. Let the CPU switch to the "double" input buffer as soon as the input buffer is empty and **asynchronously initiate an I/O operation** to reload the original input buffer.
  Treat the output buffer similarly.



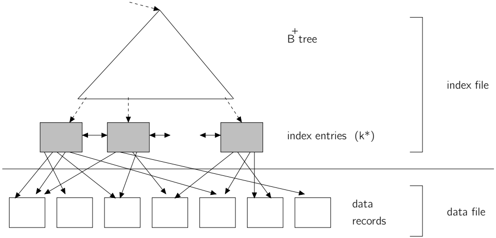
(Not) Using B$^+$-trees for Sorting

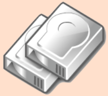
- If a B$^+$-tree matches a sorting task (*i.e.*, B$^+$-tree organized over key k with ordering θ), we *may* be better off to **access the index and abandon external sorting**.

  1. If the B$^+$-tree is **clustered**, then
     - the data file itself is already θ-sorted,

     ⇒ simply sequentially read the sequence set (or the pages of the data file).
  2. If the B$^+$-tree is **unclustered**, then
     - in the worst case, we have to initiate one I/O operation per record (not per page)!

     ⇒ do not consider the index.





(Not) Using B$^+$-tree for Sorting

- Let p denote the number of records per page (typically, $p = 10, \ldots, 1000$. Expected of I/O operations to sort via an *unclustered* B$^+$-tree will thus be $p \cdot N$:

Expected sort I/O operations (assume $B = 257$)