Chapter 5

Multi-Dimensional Indexing

Indexing Points and Regions in *k* Dimensions

Architecture and Implementation of Database Systems
Summer 2013

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More Dimensions. . .

One SQL query, many range predicates

```
SELECT *
FROM   CUSTOMERS
WHERE  ZIPCODE BETWEEN 8880 AND 8999
AND    REVENUE BETWEEN 3500 AND 6000
```

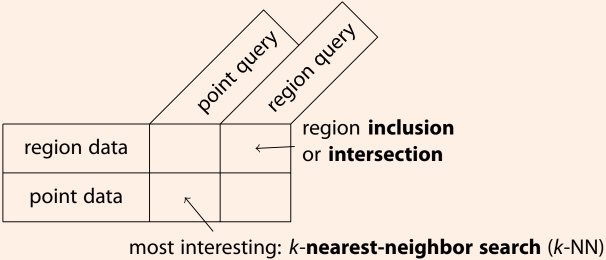
This query involves a **range predicate** in **two** dimensions.

Typical use cases for **multi-dimensional data** include

- **geographical data** (GIS: Geographical Information Systems),
- **multimedia retrieval** (*e.g.*, image or video search),
- **OLAP** (Online Analytical Processing),
- queries against **encoded data** (*e.g.*, XML)

. . . More Challenges. . .

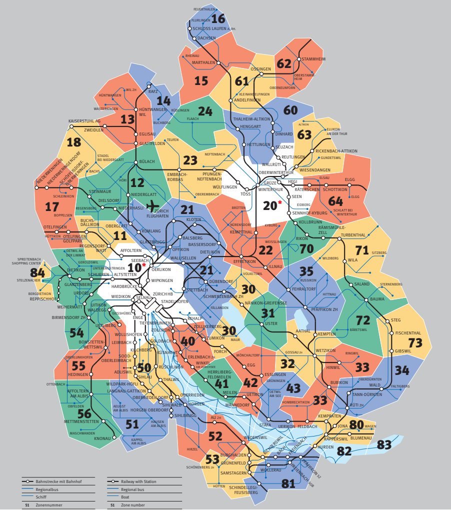
Queries and data can be points or regions

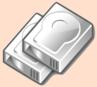


. . . and you can come up with many more meaningful types of queries over multi-dimensional data.

Note: All equality searches can be reduced to one-dimensional queries.

Points, Lines, and Regions





. . . More Solutions

Recent proposals for multi-dimension index structures

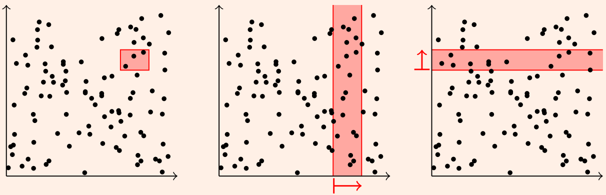
Quad Tree [Finkel 1974]	K-D-B-Tree [Robinson 1981]
R-tree [Guttman 1984]	Grid File [Nievergelt 1984]
R$^+$-tree [Sellis 1987]	LSD-tree [Henrich 1989]
R*-tree [Geckmann 1990]	hB-tree [Lomet 1990]
Vp-tree [Chiueh 1994]	TV-tree [Lin 1994]
UB-tree [Bayer 1996]	hB-$^\Pi$-tree [Evangelidis 1995]
SS-tree [White 1996]	X-tree [Berchtold 1996]
M-tree [Ciaccia 1996]	SR-tree [Katayama 1997]
Pyramid [Berchtold 1998]	Hybrid-tree [Chakrabarti 1999]
DABS-tree [Böhm 1999]	IQ-tree [Böhm 2000]
Slim-tree [Faloutsos 2000]	Landmark file [Böhm 2000]
P-Sphere-tree [Goldstein 2000]	A-tree [Sakurai 2000]

None of these provides a “fits all” solution.

Can We Just Use a B^+-tree?

- Can two B^+-trees, one over `ZIPCODE` and over `REVENUE`, adequately support a two-dimensional range query?

Querying a rectangle



- Can only scan along **either** index at once, and both of them produce many **false hits**.
- If all you have are these two indices, you can do **index intersection:** perform both scans in separation to obtain the *rids* of candidate tuples. Then compute the intersection between the two *rid* lists (DB2: `IXAND`).

IBM DB2: Index Intersection

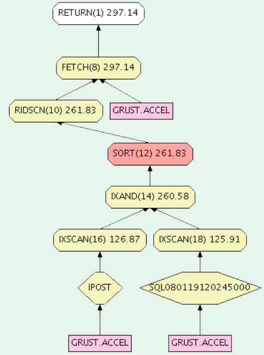
DB2 IBM DB2 uses index intersection (operator IXAND)

```
SELECT  *
FROM    ACCEL
WHERE   PRE BETWEEN 0 AND 10000
AND     POST BETWEEN 10000 AND 20000
```

Relevant indexes defined over table ACCEL:

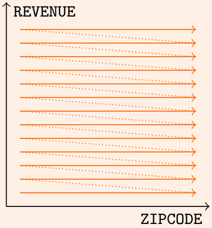
1. IPOST over column POST
2. SQL0801192024500 over column PRE (primary key)

DB2 Plan selected by the query optimizer

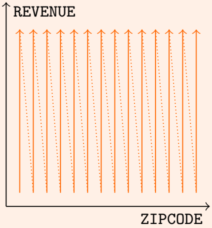


Can Composite Keys Help?

Indexes with composite keys



⟨REVENUE, ZIPCODE⟩ index



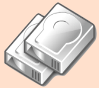
⟨ZIPCODE, REVENUE⟩ index

- **Almost the same thing!**
  Indices over composite keys are **not symmetric**: The major attribute dominates the organization of the B^+-tree.
- **But**: Since the minor attribute is also stored in the index (part of the keys *k*), we may discard non-qualifying tuples **before** fetching them from the data pages.

Multi-Dimensional Indices

- B^+-trees can answer **one-dimensional** queries **only**.[1]
- We would like to have a **multi-dimensional index structure** that
  - is **symmetric** in its dimensions,
  - **clusters** data in a space-aware fashion,
  - is **dynamic** with respect to updates, and
  - provides good support for useful queries.
- We will start with data structures that have been designed for **in-memory** use, then tweak them into **disk-aware** database indices (*e.g.*, organize data in a page-wise fashion).

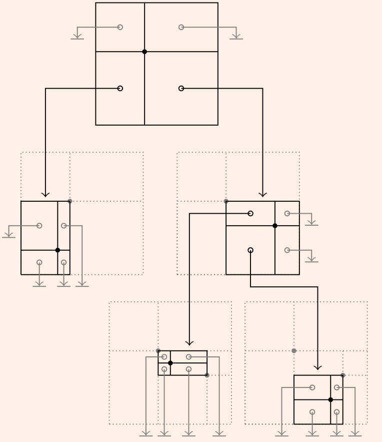
[1] Toward the end of this chapter, we will see UB-trees, a nifty trick that uses B^+-trees to support some multi-dimensional queries.



"Binary" Search Tree

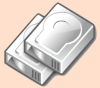
In k dimensions, a **"binary tree"** becomes a 2^k**-ary tree**.

Point quad tree ($k = 2$)



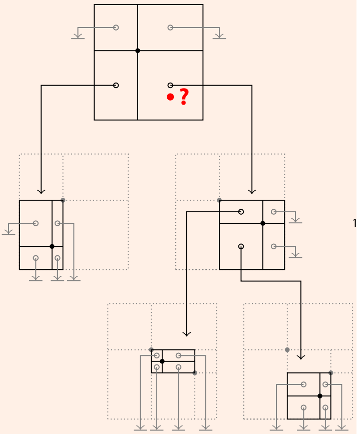
- Each data point **partitions** the data space into 2^k **disjoint regions**.
- In each node, a region points to another node (representing a refined partitioning) or to a special **null** value.
- This data structure is a **point quad tree**.

  ↗ Finkel and Bentley. Quad Trees: A Data Structure for Retrieval on Composite Keys. *Acta Informatica*, vol. 4, 1974.



Searching a Point Quad Tree

Point quad tree ($k = 2$)



Point quad tree search

Function: `p_search` (q, *node*)

if q matches data point in *node* **then**
  return data point;
else
  $P \leftarrow$ partition containing q;
  if P points to **null then**
    return not found;
  else
    $node' \leftarrow$ node pointed to by P;
    return `p_search` (q, $node'$);

Function: `pointsearch` (q)

return `p_search` (q, *root*) ;

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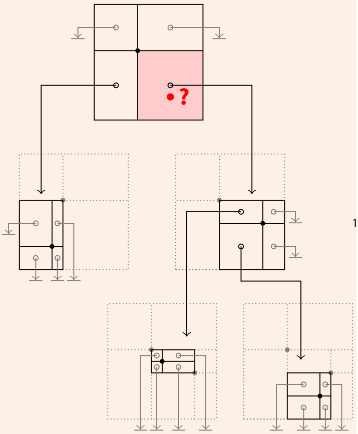

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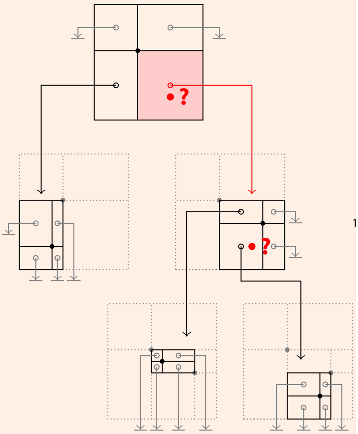

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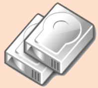

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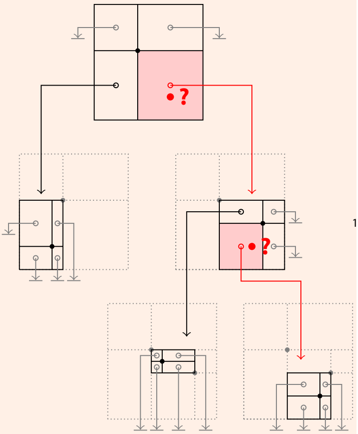

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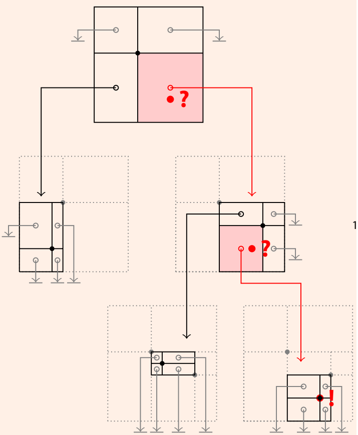

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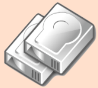


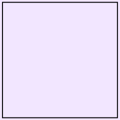
Inserting into a Point Quad Tree

Inserting a point q_{new} into a quad tree happens analogously to an insertion into a binary tree:

1. **Traverse** the tree just like during a search for q_{new} until you encounter a partition P with a **null** pointer.
2. Create a **new node** n' that spans the same area as P and is partitioned by q_{new}, with all partitions pointing to **null**.
3. Let P point to n'.

Note that this procedure does **not** keep the tree **balanced**.

Example (Insertions into an empty point quad tree)



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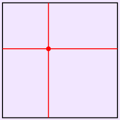

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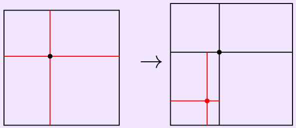

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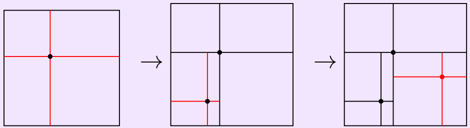

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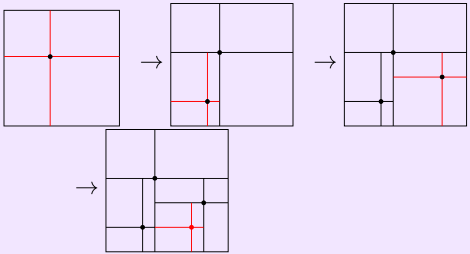

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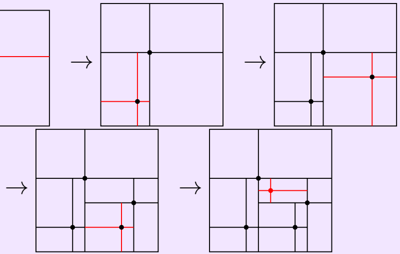


Range Queries

To evaluate a **range query**[2], we may need to follow **several** children of a quad tree node *node*:

Point quad tree range search

```
Function: r_search (Q, node)
if data point in node is in Q then
    append data point to result ;
foreach partition P in node that intersects with Q do
    node' ← node pointed to by P ;
    r_search (Q, node') ;
```

```
Function: regionsearch (Q)
return r_search (Q, root) ;
```

[2] We consider **rectangular** regions only; other shapes may be answered by querying for the **bounding rectangle** and post-processing the output.

Point Quad Trees—Discussion

Point quad trees

- ✔ are **symmetric** with respect to all dimensions and
- ✔ can answer **point queries** and **region queries**.

But

- ✗ the shape of a quad tree depends on the **insertion order** of its content, in the worst case **degenerates** into a **linked list**,
- ✗ **null** pointers are **space inefficient** (particularly for large k).

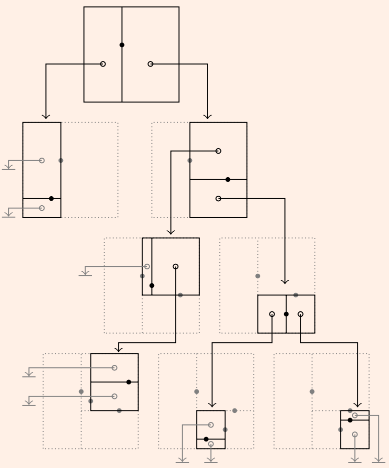
In addition,

- they can only store **point data**.

Also remember that quad trees were designed for **main memory** operation.

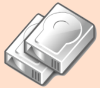
k-d Trees

Sample *k*-d tree ($k = 2$)



- Index *k*-dimensional data, but keep the tree **binary**.
- For each **tree level** l use a different **discriminator dimension** d_l along which to **partition**.
  - Typically: **round robin**
- This is a ***k*-d tree**.

  ↗ Bentley. Multidimensional Binary Search Trees Used for Associative Searching. *Comm. ACM*, vol. 18, no. 9, Sept. 1975.



k-d Trees

k-d trees inherit the positive properties of the point quad trees, but improve on **space efficiency**.

For a given point set, we can also construct a **balanced** *k*-d tree:[3]

k-d tree construction (Bentley's *OPTIMIZE* algorithm)

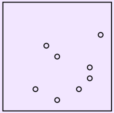
Function: `kdtree` (*pointset*, *level*)

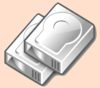
if *pointset* is empty **then**
    return null ;
else
    $p \leftarrow$ median from *pointset* (along d_{level}) ;
    $points_{\text{left}} \leftarrow \{v \in pointset \text{ where } v_{d_{level}} < p_{d_{level}}\}$;
    $points_{\text{right}} \leftarrow \{v \in pointset \text{ where } v_{d_{level}} \geq p_{d_{level}}\} \setminus \{p\}$;
    $n \leftarrow$ new *k*-d tree node, with data point p ;
    $n.left \leftarrow$ `kdtree` ($points_{\text{left}}$, $level + 1$) ;
    $n.right \leftarrow$ `kdtree` ($points_{\text{right}}$, $level + 1$) ;
    return n ;

[3] v_i: coordinate i of point v.

Balanced *k*-d Tree Construction

Example (Step-by-step balanced *k*-d tree construction)





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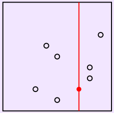

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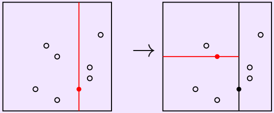

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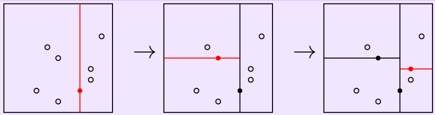

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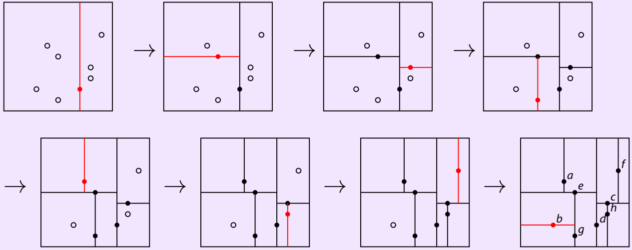

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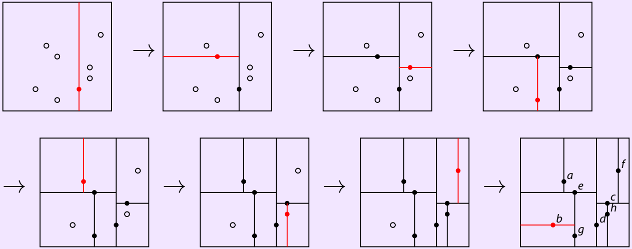
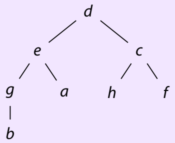


Resulting tree shape:



K-D-B-Trees

k-d trees improve on some of the deficiencies of point quad trees:

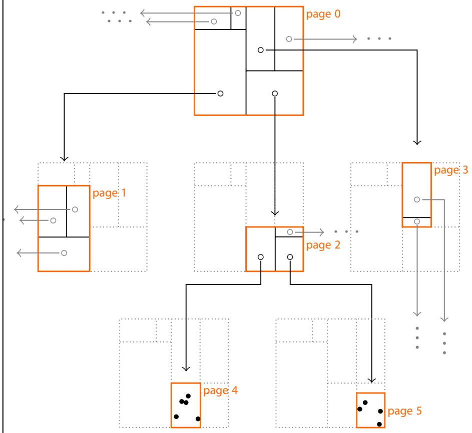
- ✔ We can **balance** a *k*-d tree by **re-building** it.
  (For a limited number of points and in-memory processing, this may be sufficient.)
- ✔ We're no longer wasting big amounts of **space**.

It's time to bring *k*-d trees to the disk. The **K-D-B-Tree**

- uses **pages** as an organizational unit
  (*e.g.*, each node in the K-D-B-tree fills a page) and
- uses a ***k*-d tree-like layout** to organize each page.

↗ John T. Robinson. The K-D-B-Tree: A Search Structure for Large Multidimensional Dynamic Indexes. *SIGMOD 1981*.

K-D-B-Tree Idea

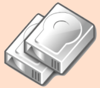


Region pages:

- contain entries $\langle region, pageID\rangle$
- no **null** pointers
- form a **balanced** tree
- all regions **disjoint** and **rectangular**

Point pages:

- contain entries $\langle point, rid\rangle$
- $\rightsquigarrow$ B^+-tree leaf nodes



K-D-B-Tree Operations

- **Searching** a K-D-B-Tree works straightforwardly:
  - Within each page determine all regions R_i that contain the query point q (intersect with the query region Q).
  - For each of the R_i, consult the page it points to and recurse.
  - On point pages, fetch and return the corresponding record for each matching data point p_i.

- When **inserting** data, we keep the K-D-B-Tree **balanced**, much like we did in the **B$^+$-tree**:
  - Simply insert a $\langle \mathit{region}, \mathit{pageID} \rangle$ ($\langle \mathit{point}, \mathit{rid} \rangle$) entry into a region page (point page) if there's **sufficient space**.
  - **Otherwise**, **split** the page.

K-D-B-Tree: Splitting a Point Page

Splitting a point page p

1. **Choose a dimension** i and an i-coordinate x_i along which to split, such that the split will result in two pages that are not overfull.
2. **Move** data points p with $p_i < x_i$ and $p_i \geqslant x_i$ to new pages p_{left} and p_{right} (respectively).
3. Replace $\langle \textit{region}, p \rangle$ in the **parent** of p with $\langle \textit{left region}, p_{\text{left}} \rangle$ $\langle \textit{right region}, p_{\text{right}} \rangle$.

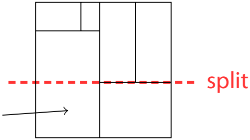
Step 3 may cause an **overflow** in p's parent and, hence, lead to a **split** of a **region page**.

K-D-B-Tree: Splitting a Region Page

- Splitting a **point page** and moving its data **points** to the resulting pages is straightforward.
- In case of a **region page split**, by contrast, some **regions** may intersect with **both** sides of the split.

Consider, *e.g.*, page 0 on slide 19:

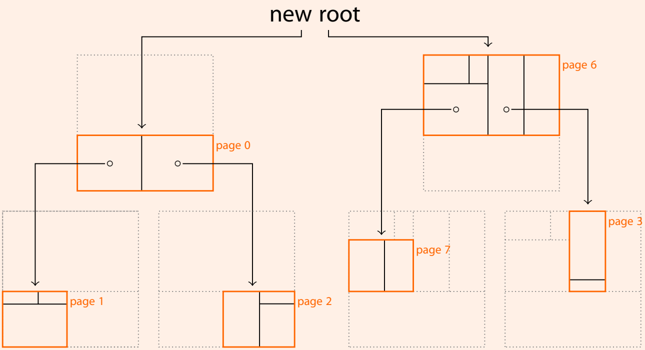
region crossing the split



- Such regions need to be **split**, too.
- This can cause a **recursive** splitting **downward** (!) the tree.

Example: Page 0 Split in K-D-B-Tree of slide 19

Split of region page 0



- Root page 0 $\Rightarrow$ pages 0 and 6 ($\rightsquigarrow$ creation of new root).
- Region page 1 $\Rightarrow$ pages 1 and 7 (point pages not shown).

K-D-B-Trees: Discussion

K-D-B-Trees

- ✔ are **symmetric** with respect to all dimensions,
- ✔ **cluster** data in a space-aware and page-oriented fashion,
- ✔ are **dynamic** with respect to updates, and
- ✔ can answer **point queries** and **region queries**.

However,

- we still don't have support for **region data** and
- K-D-B-Trees (like *k*-d trees) will not handle **deletes** dynamically.

This is because we always partitioned the data space such that

- every region is **rectangular** and
- regions never **intersect**.

R-Trees

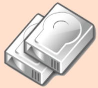
R-trees do not have the disjointness requirement:

- R-tree inner or leaf nodes contain $\langle \mathit{region}, \mathit{pageID} \rangle$ or $\langle \mathit{region}, \mathit{rid} \rangle$ entries (respectively).
  region is the **minimum bounding rectangle** that spans all data items reachable by the respective pointer.
- Every node contains between d and $2d$ entries ($\rightsquigarrow$ B$^+$-tree).[4]
- **Insertion** and **deletion** algorithms keep an R-tree **balanced** at all times.

R-trees allow the storage of **point and region data**.

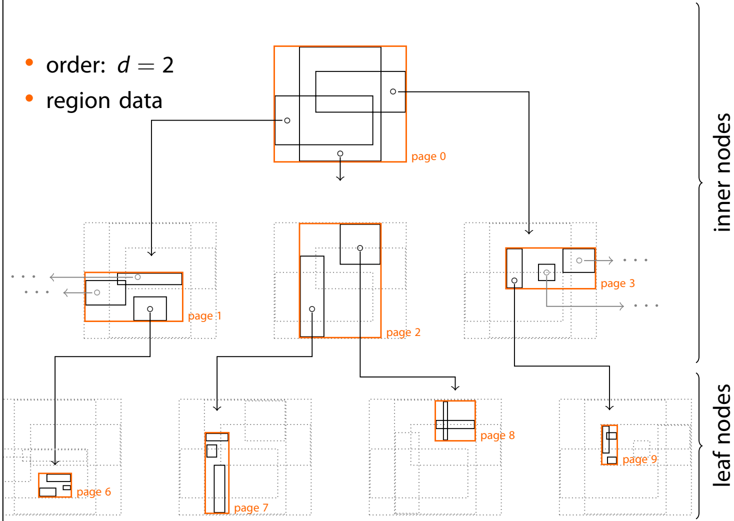
↗ Antonin Guttman. R-Trees: A Dynamic Index Structure for Spatial Searching. *SIGMOD 1984*.

[4] except the root node



R-Tree: Example

- order: $d = 2$
- region data



R-Tree: Searching and Inserting

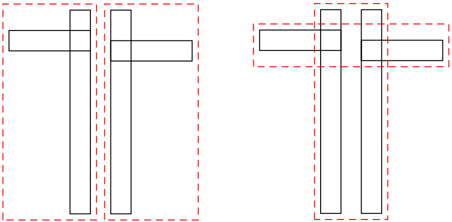
While **searching** an R-tree, we may have to descend into more than one child node for point **and** region queries (↗ range search in point quad trees, slide 13).

Inserting into an R-tree (cf. B^+-tree insertion)

1. **Choose** a leaf node *n* to insert the new entry.
   - Try to minimize the necessary region enlargement(s).
2. If *n* is **full**, **split** it (resulting in *n* and *n*′) and distribute old and new entries evenly across *n* and *n*′.
   - Splits may propagate bottom-up and eventually reach the root (↗ B^+-tree).
3. After the insertion, some regions in the ancestor nodes of *n* may need to be **adjusted** to cover the new entry.

Splitting an R-Tree Node

To **split** an R-tree node, we generally have more than one alternative:



Heuristic: Minimize the totally covered area. But:

- **Exhaustive** search for the best split is infeasible.
- Guttman proposes two ways to **approximate** the search.
- Follow-up papers (*e.g.*, the R*-tree) aim at improving the quality of node splits.

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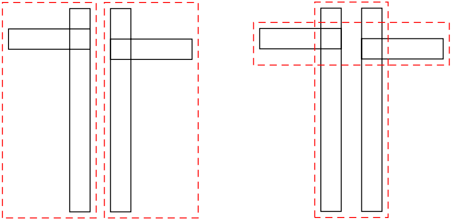

“bad” split   “good” split

Heuristic: Minimize the totally covered area. But:

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R-Tree: Deletes

All R-tree invariants (see 25) are maintained during **deletions**.

1. If an R-tree node *n* **underflows** (*i.e.*, less than *d* entries are left after a deletion), the whole node is **deleted**.
2. Then, all entries that existed in *n* are **re-inserted** into the R-tree.

Note that Step 1 may lead to a recursive deletion of *n*'s parent.

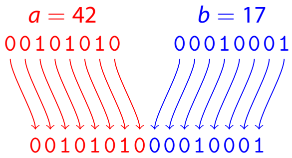
- Deletion, therefore, is a rather **expensive** task in an R-tree.

Spatial indexing in mainstream database implementations

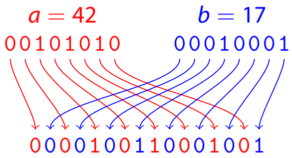
- Indexing in commercial database systems is typically based on **R-trees**.
- Yet, only few systems implement them out of the box (*e.g.*, PostgreSQL). Most require the licensing/use of specific extensions.

Bit Interleaving

- We saw earlier that a B$^+$-tree over **concatenated** fields $\langle a, b\rangle$ does not help our case, because of the **asymmetry** between the role of a and b in the index key.
- What happens if we **interleave** the bits of a and b (hence, make the B$^+$-tree "more symmetric")?

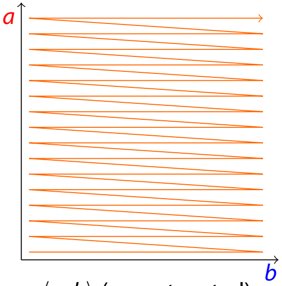


$\langle a, b\rangle$ (concatenation)

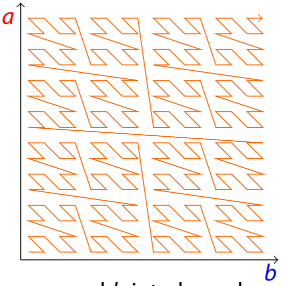


a and b interleaved

Z-Ordering



$\langle a, b \rangle$ (concatenated)



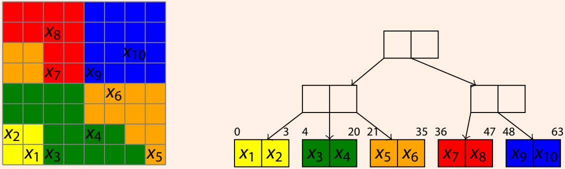
a and *b* interleaved

- Both approaches **linearize** all coordinates in the value space according to some **order**. ↗ see also slide 8
- Bit interleaving leads to what is called the **Z-order**.
- The Z-order (largely) preserves spatial **clustering**.

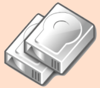
UB-Tree: B^+-trees Over Z-Codes

- Use a **B^+-tree** to index Z-codes of multi-dimensional data.
- Each leaf in the B^+-tree describes an **interval** in the **Z-space**.
- Each interval in the Z-space describes a **region** in the multi-dimensional data space:

UB-Tree: B^+-tree over Z-codes

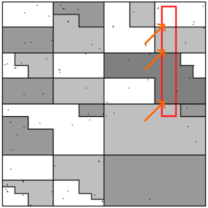


- To retrieve all data points in a query region Q, try to touch only those leave pages that **intersect** with Q.



UB-Tree: Range Queries

After each page processed, perform an **index re-scan** (↗) to fetch the next leaf page that intersects with Q.



Example by Volker Markl and Rudolf Bayer, taken from http://mistral.in.tum.de/

UB-tree range search (Function `ub_range(Q)`)

```
cur ← z(Q_bottom,left) ;
while true do
    // search B+-tree page containing cur (↗ slide 0.0)
    page ← search (cur);
    foreach data point p on page do
        if p is in Q then
            append p to result ;
    if region in page reaches beyond Q_top,right then
        break ;
    // compute next Z-address using Q and data on current
       page
    cur ← get_next_z_address (Q, page);
```

UB-Trees: Discussion

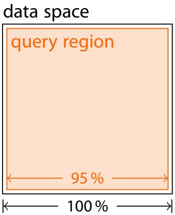
- Routine `get_next_z_address()` (return next Z-address lying in the query region) is non-trivial and depends on the shape of the Z-region.
- UB-trees are **fully dynamic**, a property inherited from the underlying B^+-trees.
- The use of other **space-filling curves** to linearize the data space is discussed in the literature, too. *E.g.*, **Hilbert curves**.

UB-trees have been commercialized in the Transbase® database system.

Spaces with High Dimensionality

For large k, all techniques that have been discussed become **ineffective**:

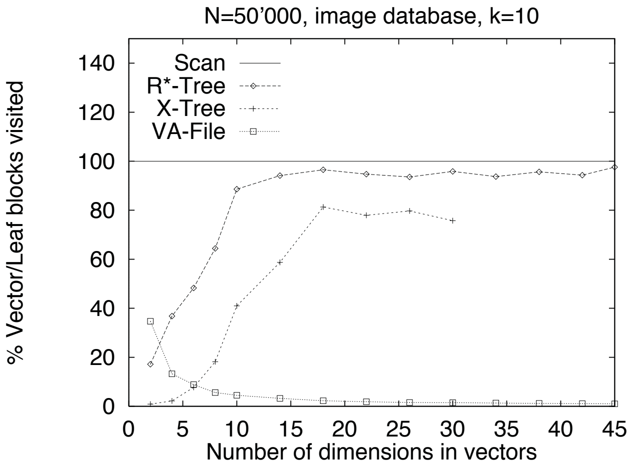
- For example, for $k = 100$, we get $2^{100} \approx 10^{30}$ partitions per node in a **point quad tree**. Even with billions of data points, **almost all** of these are empty.
- Consider a **really big** search region, cube-sized covering 95 % of the range along **each** dimension:



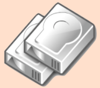
For $k = 100$, the probability of a point being in this region is still only $0.95^{100} \approx 0.59\,\%$.

- We experience the **curse of dimensionality** here.

Page Selectivty for *k*-NN Search



Data: Stephen Bloch. What’s Wrong with High-Dimensionality Search. *VLDB 2008*.



Wrap-Up

Point Quad Tree

k-dimensional analogy to binary trees; main memory only.

k-d Tree, K-D-B-Tree

k-d tree: partition space one dimension at a time (round-robin); K-D-B-Tree: B^+-tree-like organization with pages as nodes, nodes use a k-d-like structure internally.

R-Tree

regions within a node may overlap; fully dynamic; for point and region data.

UB-Tree

use space-filling curve (Z-order) to linearize k-dimensional data, then use B^+-tree.

Curse Of Dimensionality

most indexing structures become ineffective for large k; fall back to sequential scanning and approximation/compression.