Chapter 3
Indexing
Navigate and Search Large Data Volumes

Architecture and Implementation of Database Systems
Summer 2013

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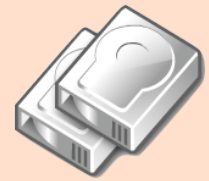


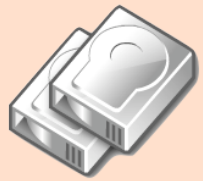
File Organization and Indexes

- A **heap file** provides just enough structure to maintain a collection of records (of a table).
- The heap file supports **sequential scans** (`openScan(·)`) over the collection, *e.g.*

SQL query leading to a sequential scan

```
1 SELECT A,B
2 FROM   R
```

But: No further operations receive specific support from the heap file.



File Organization and Indexes

- For queries of the following type, it would definitely be helpful if the SQL query processor could rely on a particular **file organization** of the records in the file for table `R`:

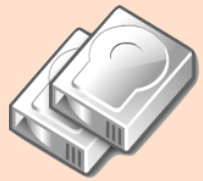
Queries calling for systematic file organization

```
1 SELECT A,B
2 FROM   R
3 WHERE  C > 42
```

```
1 SELECT   A,B
2 FROM     R
3 ORDER BY C ASC
```

✎ File organization for table `R`

Which organization of records in the file for table `R` could speed up the evaluation of **both** queries above?



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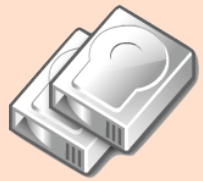
✎ File organization for table `R`

Which organization of records in the file for table `R` could speed up the evaluation of **both** queries above?
Allocate records of table `R` in **ascending order of** `C` **attribute values**, place records on neighboring pages. (Only include columns `A`, `B`, `C` in records.)



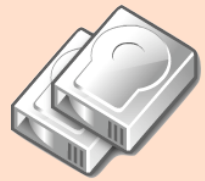
A Competition of File Organizations

- This chapter presents a comparison of three **file organizations**:
  1. Files of **randomly ordered** records (heap files)
  2. Files **sorted** on some record field(s)
  3. Files **hashed** on some record field(s)
- A file organization is tuned to make a certain query (class) efficient, but if we have to support **more than one query class**, we may be in trouble. Consider:

Query Q calling for organization sorted on column `A`

```
1 SELECT A,B,C
2 FROM   R
3 WHERE  A > 0 AND A < 100
```

- If the file for table `R` is sorted on `C`, this does not buy us anything for query Q.
- If Q is an important query but is *not* supported by `R`'s file organization, we can build a support data structure, an **index**, to speed up (queries similar to) Q.



A Competition of File Organizations

- This competition assesses 3 file organizations in 5 disciplines:
  1. **Scan**: read all records in a given file.
  2. **Search with equality test**

Query calling for equality test support

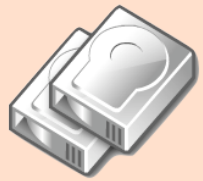
```
1 SELECT *
2 FROM   R
3 WHERE  C = 42
```

  3. **Search with range selection** (upper or lower bound might be unspecified)

Query calling for range selection support

```
1 SELECT *
2 FROM   R
3 WHERE  A > 0 AND A < 100
```

  4. **Insert** a given record in the file, respecting the file's organization.
  5. **Delete** a record (identified by its *rid*), maintain the file's organization.

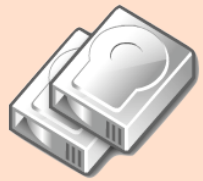


Simple Cost Model

- Performing these 5 database operations clearly involves **block I/O**, the major cost factor.
- However, we have to additionally pay for CPU time used to **search inside a page**, **compare a record field to a selection constant**, *etc*.
- To analyze cost more accurately, we introduce the following parameters:

Simple cost model parameters

Parameter	Description
b	# of pages in the file
r	# of records on a page
D	time needed to read/write a **d**isk page
C	**C**PU time needed to process a record (*e.g.*, compare a field value)
H	CPU time taken to apply a **h**ash function to a record



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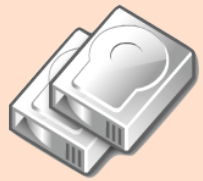
D	time needed to read/write a **d**isk page
C	**C**PU time needed to process a record (*e.g.*, compare a field value)
H	CPU time taken to apply a function to a record (*e.g.*, a comparison or **h**ash function)

Remarks:

- $D \approx 15\,\text{ms}$
- $C \approx H \approx 0.1\,\mu\text{s}$
- This is a coarse model to **estimate** the actual execution time (this does *not* model network access, cache effects, burst I/O, …).



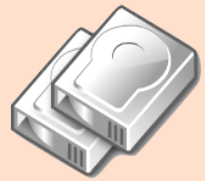
Aside: Hashing

A simple hash function

A **hashed file** uses a **hash function** h to map a given record onto a specific page of the file.

Example: h uses the lower 3 bits of the first field (of type `INTEGER`) of the record to compute the corresponding page number:

$$
\begin{array}{lcll}
h\left(\langle 42, \texttt{true}, \texttt{'foo'} \rangle\right) & \rightarrow & 2 & (42 = 101010_2) \\
h\left(\langle 14, \texttt{true}, \texttt{'bar'} \rangle\right) & \rightarrow & 6 & (14 = 1110_2) \\
h\left(\langle 26, \texttt{false}, \texttt{'baz'} \rangle\right) & \rightarrow & 2 & (26 = 11010_2)
\end{array}
$$



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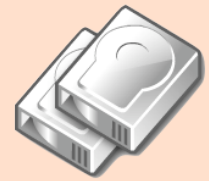
\end{array}
$$

- The hash function determines the page number only; record placement inside a page is not prescribed.
- If a page p is filled to capacity, a chain of **overflow** pages is maintained to store additional records with $h\left(\langle \ldots \rangle\right) = p$.
- To avoid immediate overflowing when a new record is inserted, pages are typically filled to 80 % only when a heap file is initially (re)organized as a hashed file.



Scan Cost

1. **Heap file**
   Scanning the records of a file involves **reading all b pages** as well as **processing each of the r records on each page**:

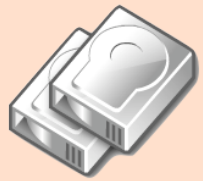
$$\text{Scan}_{\text{heap}} = b \cdot (D + r \cdot C)$$

2. **Sorted file**
   The sort order does not help much here. However, the scan retrieves the records in sorted order (which can be big plus later on):

$$\text{Scan}_{\text{sort}} = b \cdot (D + r \cdot C)$$

3. **Hashed file**
   Again, the hash function does not help. We simply scan from the beginning (skipping over the spare free space typically found in hashed files):

$$\text{Scan}_{\text{hash}} = \underbrace{(100/80)}_{=1.25} \cdot b \cdot (D + r \cdot C)$$

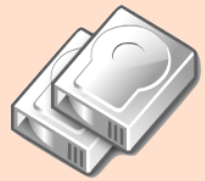


Cost for Search With Equality Test (`A` = *const*)

1. **Heap file**
   Consider (a) the equality test is on a *primary key*, (b) the equality test is *not* on a *primary key*:

   $$\text{(a)} \quad \text{Search}_{\text{heap}} = 1/2 \cdot b \cdot (D + r \cdot (C + H))$$
   $$\text{(b)} \quad \text{Search}_{\text{heap}} = b \cdot (D + r \cdot (C + H))$$

2. **Sorted file** (sorted on `A`)
   We assume the equality test to be on the field determining the sort order. The sort order enables us to use **binary search**:



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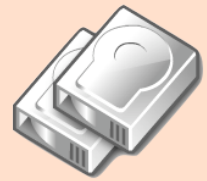
2. **Sorted file** (sorted on `A`)
   We assume the equality test to be on the field determining the sort order. The sort order enables us to use **binary search**:

   $$\text{Search}_{\text{sort}} = \log_2 b \cdot (D + \log_2 r \cdot (C + H))$$

   If more than one record qualifies, all other matches are stored right after the first hit.

   (Nevertheless, **no DBMS** will implement binary search for value lookup.)



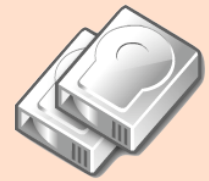
Cost for Search With Equality Test (`A` = *const*)

3. **Hashed file** (hashed on `A`)
   Hashed files support equality searching best. The hash function directly leads us to the page containing the hit (overflow chains ignored here).

   Consider (a) the equality test is on a *primary key*, (b) the equality test is *not* on a *primary key*:

$$\text{(a)} \quad \text{Search}_{\text{hash}} = H + D + {}^1\!/_2 \cdot r \cdot C$$
$$\text{(b)} \quad \text{Search}_{\text{hash}} = H + D + r \cdot C$$

   No dependence on file size b here. (All qualifying records live on the same page or, if present, in its overflow chain.)



Cost for Range Selection (`A >=` *lower* `AND A <=` *upper*)

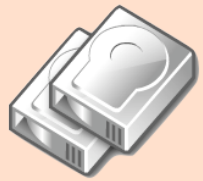
1. **Heap file**
   Qualifying records can appear anywhere in the file:

$$\text{Range}_{\text{heap}} = b \cdot (D + r \cdot (C + 2 \cdot H))$$

2. **Sorted file** (sorted on `A`)
   Use equality search (with `A =` *lower*), **then sequentially scan** the file until a record with `A >` *upper* is encountered:

$$\text{Range}_{\text{sort}} = \log_2 b \cdot (D + \log_2 r \cdot (C + H)) + \lceil n/r \rceil \cdot D + n \cdot (C + H)$$

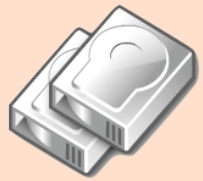
($n + 1$ overall hits in the range, $n \geq 0$)



Cost for Range Selection (`A >=` *lower* `AND A <=` *upper*)

3. **Hashed file** (hashed on `A`)
   Hashing offers no help here as hash functions are designed to **scatter** records all over the hashed file (*e.g.*, for the h we considered earlier: $h(\langle 7, \ldots \rangle) = 7, h(\langle 8, \ldots \rangle) = 0$):

$$\text{Range}_{\text{hash}} = (100/80) \cdot b \cdot (D + r \cdot (C + H))$$



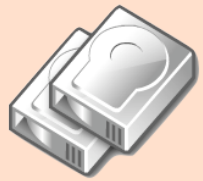
Insertion Cost

1. **Heap file**
   We can add the record to some **arbitrary** free page (finding that page is not accounted for here).
   This involves reading and writing that page:

$$\text{Insert}_{\text{heap}} = 2 \cdot D + C$$

2. **Sorted file**
   On average, the new record will belong in the middle of the file. After insertion, we have to **shift all subsequent records** (in the second half of the file):

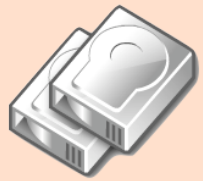
$$\text{Insert}_{\text{sort}} = \log_2 b \cdot (D + \log_2 r \cdot (C + H)) + 1/2 \cdot b \cdot (\underbrace{D + r \cdot C + D}_{\text{read + shift + write}})$$



Insertion Cost

3. **Hashed file**
   We pretend to search for the record, then read and write the page determined by the hash function (here we assume the spare 20 % space on the page is sufficient to hold the new record):

$$\text{Insert}_{\text{hash}} = \underbrace{H + D}_{\text{search}} + C + D$$



Deletion Cost (Record Specified by its *rid*)

1. **Heap file**
   If we do not try to compact the file after we have found and removed the record (because the file uses free space management), the cost is:

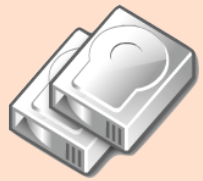
$$\text{Delete}_{\text{heap}} = \underbrace{D}_{\text{search by } rid} + C + D$$

2. **Sorted file**
   Again, we access the record's page and then (on average) shift the second half the file to compact the file:

$$\text{Delete}_{\text{sort}} = D + 1/2 \cdot b \cdot (D + r \cdot C + D)$$

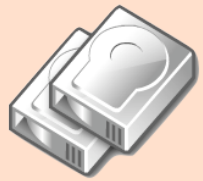
3. **Hashed file**
   Accessing the page using the *rid* is even faster than the hash function, so the hashed file behaves like the heap file:

$$\text{Delete}_{\text{hash}} = D + C + D$$



The “Best” File Organization?

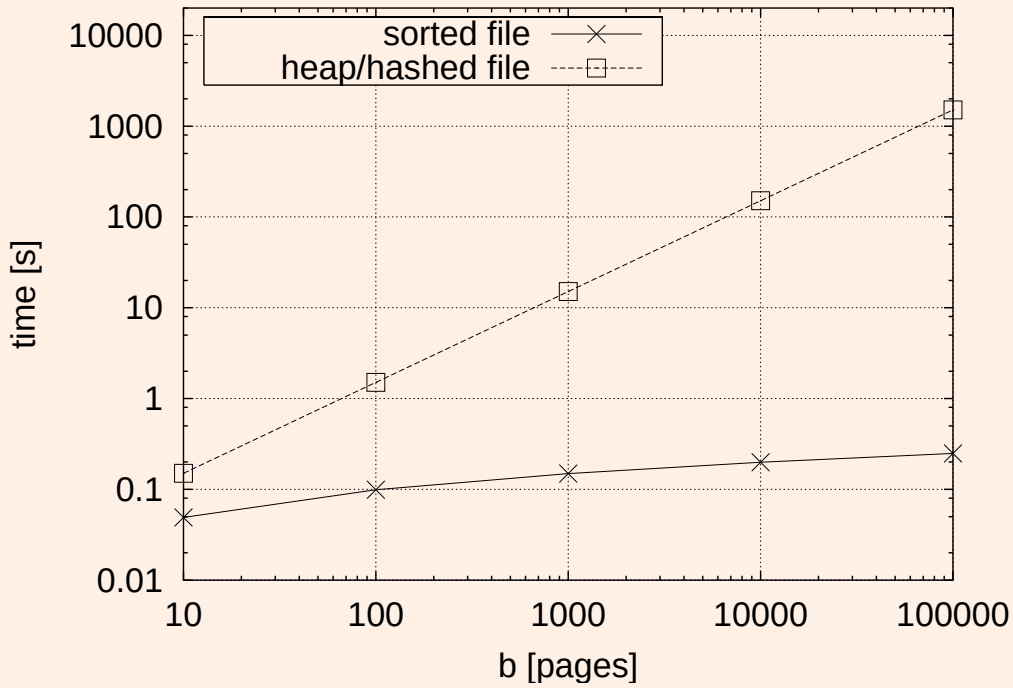
- There is **no single file organization** that responds equally fast to all 5 operations.
- This is a dilemma, because more advanced file organizations can really make a difference in speed.

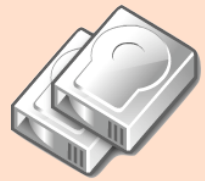


Range selection performance

- Performance of **range selections** for files of increasing size ($D = 15\,\text{ms}, C = 0.1\,\mu\text{s}, r = 100, n = 10$):

Range Selection Performance

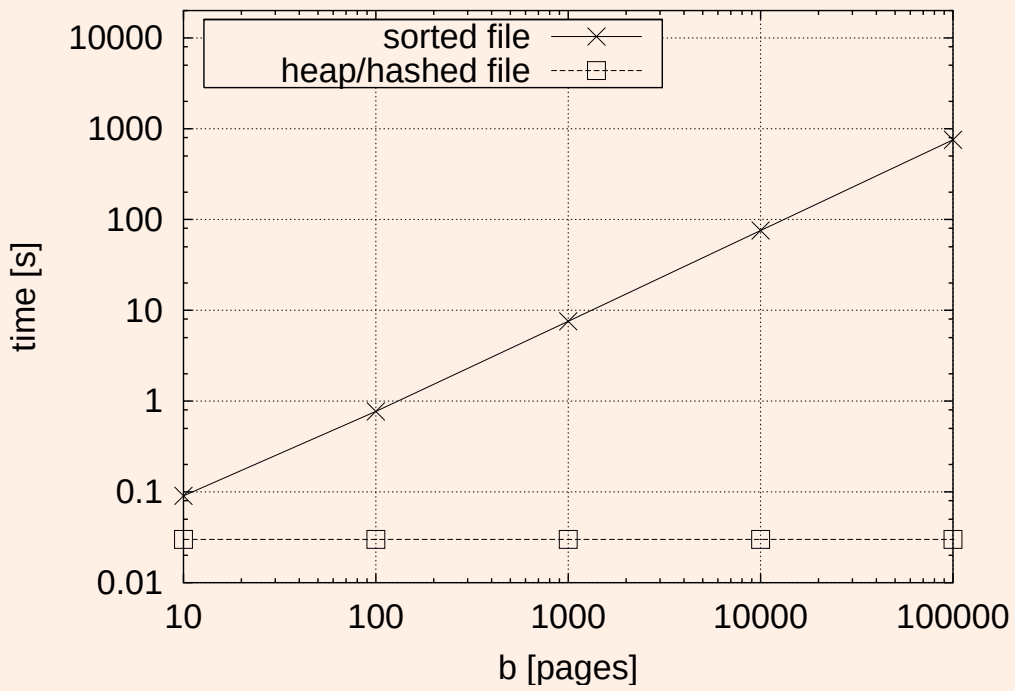


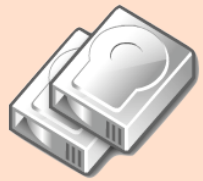


Deletion Performance Comparison

- Performance of **deletions** for files of increasing size ($D = 15\,\text{ms}, C = 0.1\,\mu\text{s}, r = 100$):

Deletion Performance

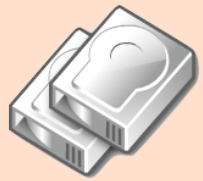




Indexes

- There exist **index structures** which offer all the advantages of a sorted file *and* support insertions/deletions efficiently[1]: **B^+-trees**.
- Before we turn to B^+-trees in detail, the following sheds some light on indexes in general.

[1] At the cost of a modest space overhead.

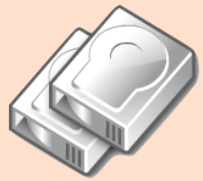


Indexes

- If the basic organization of a file does not support a particular operation, we can **additionally maintain** an auxiliary structure, an **index**, which adds the needed support.
- We will use indexes like **guides**. Each guide is specialized to accelerate searches on a specific attribute `A` (or a combination of attributes) of the records in its associated file:

Index usage

1. Query the index for the location of a record with `A` = k (k is the **search key**).
2. The index responds with an associated **index entry** $k*$ ($k*$ contains sufficient information to access the actual record in the file).



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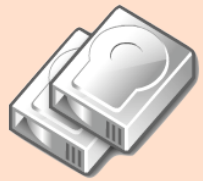
2. The index responds with an associated **index entry** $k*$ ($k*$ contains sufficient information to access the actual record in the file).
3. Read the actual record by using the guiding information in $k*$; the record will have an `A`-field with value k.

$$k \underbrace{\longrightarrow}_{\text{index access}} k* \underbrace{\longrightarrow}_{\text{file access}} \langle \ldots, \texttt{A} = k, \ldots \rangle$$

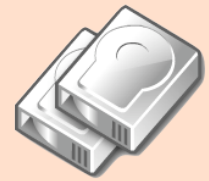


Index entries

- We can design the **index entries**, *i.e.*, the $k*$, in various ways:

Index entry designs

Variant	Index entry $k*$
A	$\langle k, \quad \langle \ldots, \mathtt{A} = k, \ldots \rangle \rangle$



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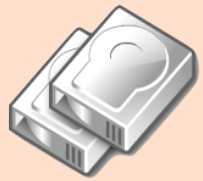
B	$\langle k, \quad rid \rangle$



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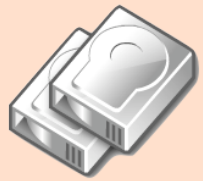
A	$\langle k, \langle \ldots, \texttt{A} = k, \ldots \rangle \rangle$
B	$\langle k, rid \rangle$
C	$\langle k, [rid_1, rid_2, \ldots] \rangle$



Index entries

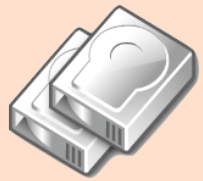
- We can design the **index entries**, *i.e.*, the $k*$, in various ways:

Index entry designs

Variant	Index entry $k*$
A	$\langle k, \quad \langle \ldots, \mathtt{A} = k, \ldots \rangle \rangle$
B	$\langle k, \quad rid \rangle$
C	$\langle k, \quad [rid_1, rid_2, \ldots] \rangle$

Remarks:

- With variant A, there is *no* need to store the data records in addition to the index—the index itself is a special file organization.



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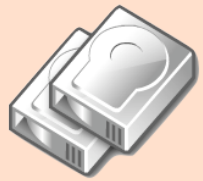
- If we build multiple indexes for a file, at most one of these should use variant A



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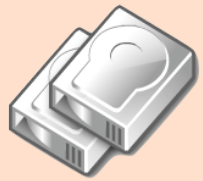

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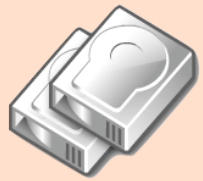
Remarks:

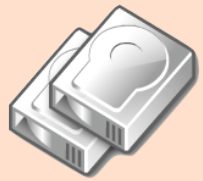
- With variant A, there is *no* need to store the data records in addition to the index—the index itself is a special file organization.
- If we build multiple indexes for a file, at most one of these should use variant A to avoid redundant storage of records.
- Variants B and C use *rid*(s) to point into the actual data file.
- Variant C leads to less index entries if multiple records match a search key k, but index entries are of variable length.



Indexing Example

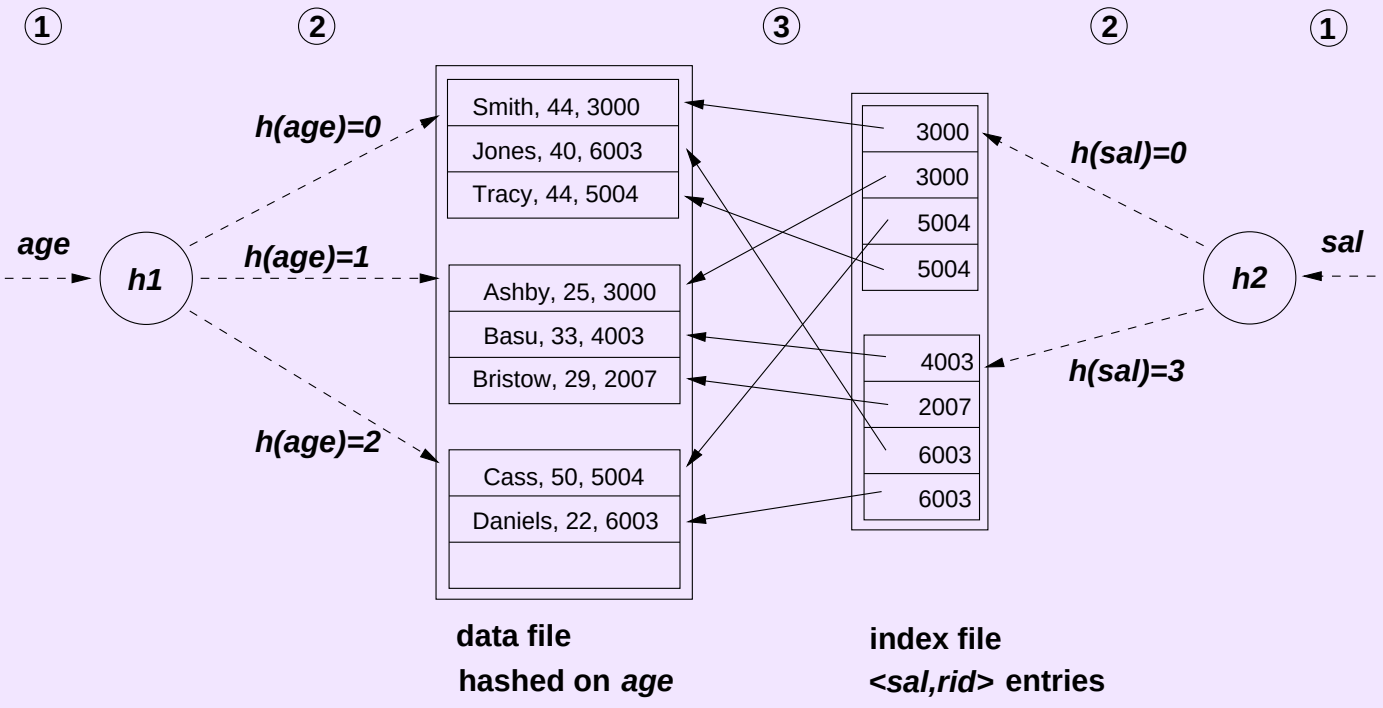
Example (Indexing example, see following slide)

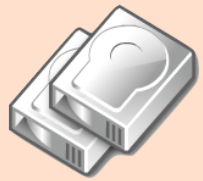
- The data file contains $\langle$`name`, `age`, `sal`$\rangle$ records of table `employees`, the file itself (index entry variant A) is hashed on field *age* (hash function *h1*).
- The index file contains $\langle$`sal`, `rid`$\rangle$ index entries (variant B), pointing into the data file.
- This file organization + index efficiently supports equality searches on the `age` **and** `sal` keys.



Indexing Example

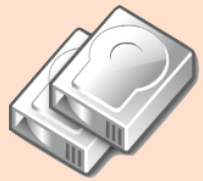
Example (Indexing example)





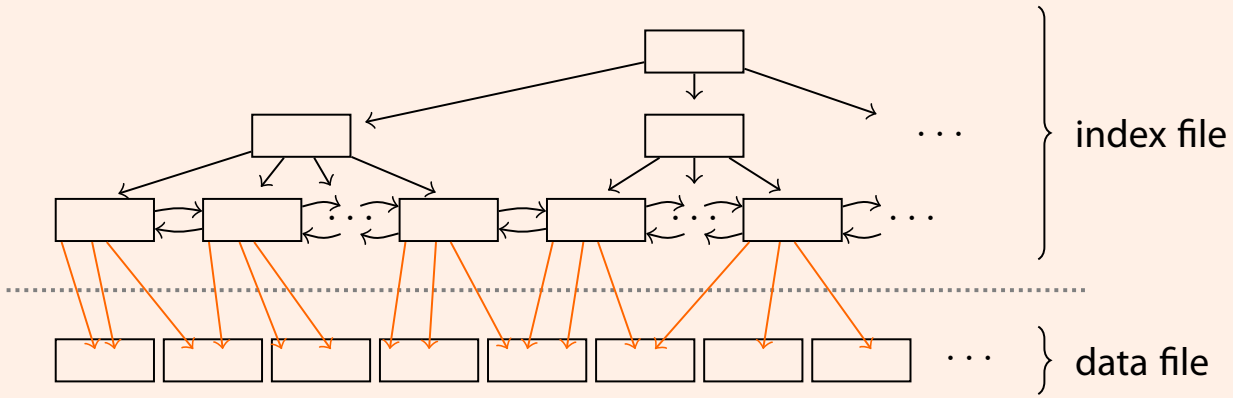
Index Properties: Clustered vs. Unclustered

- Suppose, we have to support **range selections** on records such that `A` >= *lower* `AND A` <= *upper*.
- If we maintain an index on the `A`-field, we can
  1. **query the index** *once* for a record with `A` = *lower*, and then
  2. **sequentially scan the data file** from there until we encounter a record with field `A` > *upper*.
- **However:** This switch from index to data file will only work provided that the **data file itself is sorted on the field** `A`.



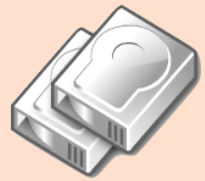
Index Properties: Clustered vs. Unclustered

Index over a data file with matching sort order

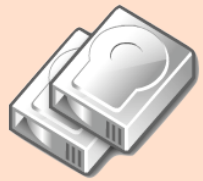


Remark:

- In a B^+-tree, for example, the index entries $k*$ stored in the leaves are sorted by key k.



Index Properties: Clustered vs. Unclustered



Definition (Clustered index)

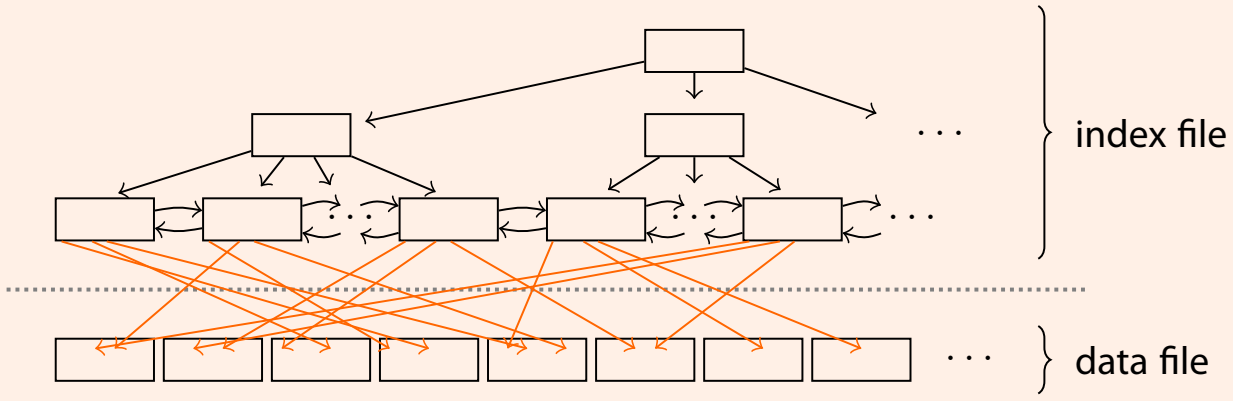
If the **data file** associated with an index **is sorted on the index search key**, the index is said to be **clustered**.

In general, the cost for a range selection grows tremendously if the index on `A` is **unclustered**. In this case, proximity of index entries does *not* imply proximity in the data file:

- As before, we can query the index for a record with `A` = *lower*.
  To continue the scan, however, we have to **revisit the index entries** which point us to **data pages scattered** all over the data file.

Index Properties: Clustered vs. Unclustered

Unclustered index



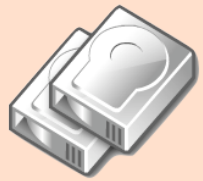
Remarks:

- If an index uses entries $k*$ of variant Ⓐ, the index is obviously clustered by definition.

Variant Ⓐ in Oracle 8i

```
CREATE TABLE ... (... PRIMARY KEY (...)) ORGANIZATION INDEX;
```

- A data file can have at most one clustered index (but any number of unclustered indexes).



Index Properties: Clustered vs. Unclustered

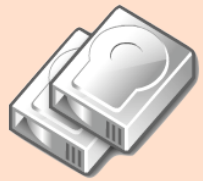
DB2 Clustered indexes

Create a clustered index `IXR` on table `R`, index key is attribute `A`:

```
CREATE INDEX IXR ON R(A ASC) CLUSTER
```

The DB2 V9.5 manual says:

> *"[`CLUSTER`] specifies that the index is **the** clustering index of the table. The **cluster factor** of a clustering index is maintained or improved dynamically as data is inserted into the associated table, by attempting to **insert new rows physically close to the rows for which the key values of this index are in the same range**. Only one clustering index may exist for a table so `CLUSTER` may not be specified if it was used in the definition of any existing index on the table (SQLSTATE 55012). A clustering index may not be created on a table that is defined to use append mode (SQLSTATE 428D8)."*



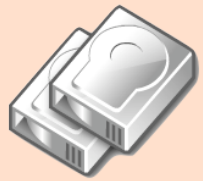
Index Properties: Clustered vs. Unclustered

Cluster a table based on an existing index

Reorganize rows of table `R` so that their physical order matches the *existing* index `IXR`:

```
CLUSTER R USING IXR
```

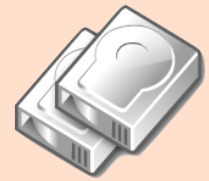
- If `IXR` indexes attribute `A` of `R`, rows will be sorted in ascending `A` order.
- The evaluation of range queries will **touch less pages** (which additionally, will be physically adjacent).
- **Note:** Generally, future insertions will compromise the perfect `A` order.
  - May issue `CLUSTER R` again to re-cluster.
  - In `CREATE TABLE`, use `WITH (fillfactor = f)`, $f \in 10 \ldots 100$, to reserve page space for subsequent insertions.



Index Properties: Clustered vs. Unclustered

DB2 Inspect clustered index details

```
1  db2 => reorgchk update statistics on table grust.accel
2
3  Doing RUNSTATS ...
4
5  Table statistics:
   :
6  :
7  Index statistics:
8
9  F4: CLUSTERRATIO or normalized CLUSTERFACTOR > 80
10
11 SCHEMA.NAME                  INDCARD  LEAF LVLS    KEYS  F4  REORG
12 ------------------------------------------------------------------
13 Table: GRUST.ACCEL
14 Index: GRUST.IKIND            235052   529    3       3  99  -----
15 Index: GRUST.IPAR             235052   675    3   67988  99  -----
16 Index: GRUST.IPOST            235052   980    3  235052 100  -----
17 Index: GRUST.ITAG             235052   535    3      75  80  *----
18 Index: SYSIBM.SQL080119120245000
19                               235052   980    3  235052 100  -----
20 ------------------------------------------------------------------
```

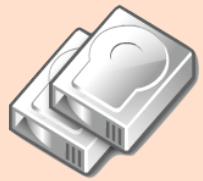
Index Properties: Dense vs. Sparse

A **clustered index** comes with more advantages than the improved performance for range selections. We can additionally design the index to be **space efficient**:

Definition (Sparse index)

To keep the size of the index small, maintain **one index entry** $k*$ **per data file page** (not one index entry per data record). Key k is the **smallest key** on that page.

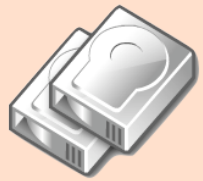
Indexes of this kind are called **sparse**. (Otherwise, indexes are referred to as **dense**.)



Index Properties: Dense vs. Sparse

To search a record with field $\texttt{A} = k$ in a sparse $\texttt{A}$-index,

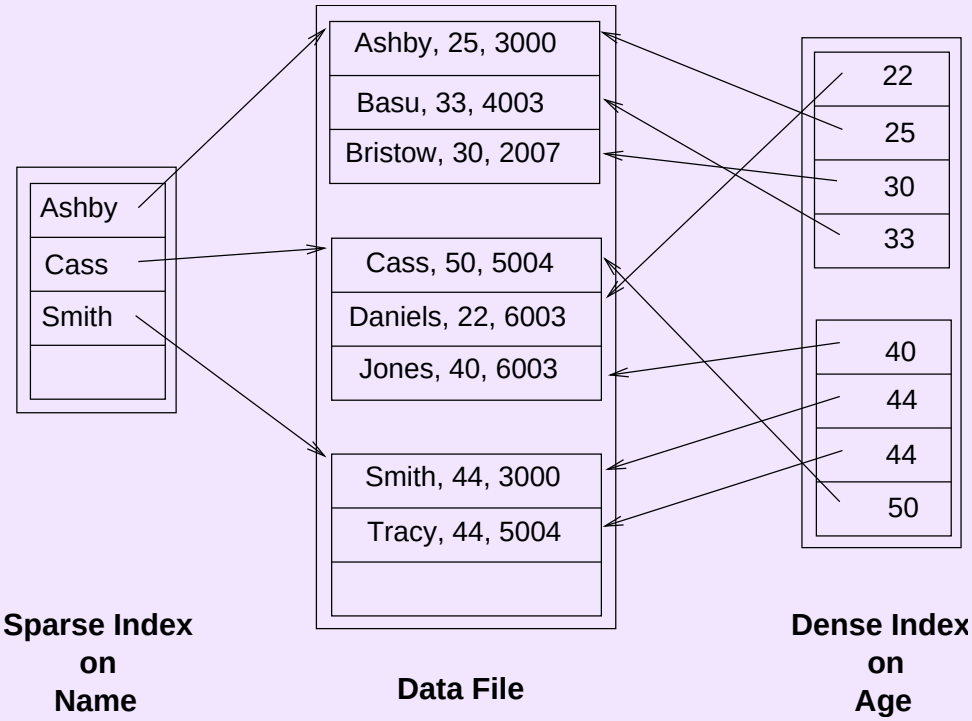
1. Locate the largest index entry $k'*$ such that $k' \leqslant k$, then
2. access the page pointed to by $k'*$, and
3. scan this page (and the following pages, if needed) to find records with $\langle \ldots, \texttt{A} = k, \ldots \rangle$.
   Since the data file is clustered (*i.e.*, sorted) on field $\texttt{A}$, we are guaranteed to find matching records in the proximity.

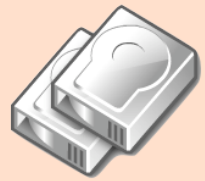


Index example (Dense vs. Sparse)

Example (Sparse index example)

- Again, the data file contains ⟨`name`, `age`, `sal`⟩ records. We maintain a **clustered sparse index** on field `name` and an **unclustered dense index** on field `age`. Both use index entry variant Ⓑ to point into the data file.





Index Properties: Dense vs. Sparse

Note:

- Sparse indexes need 2–3 orders of magnitude less space than dense indexes (consider # records/page).
- We *cannot* build a sparse index that is unclustered (*i.e.*, there is at most one sparse index per file).

✎ SQL queries and index exploitation

How do you propose to evaluate the following SQL queries?

```
1 SELECT MAX(age)
2 FROM   employees
```

```
1 SELECT MAX(name)
2 FROM   employees
```