Chapter 5

Multi-Dimensional Indexing

Indexing Points and Regions in *k* Dimensions

Architecture and Implementation of Database Systems
Winter 2010/11

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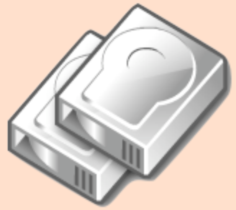


More Dimensions. . .

One SQL query, many range predicates

```
SELECT *
FROM   CUSTOMERS
WHERE  ZIPCODE BETWEEN 8880 AND 8999
AND    REVENUE BETWEEN 3500 AND 6000
```

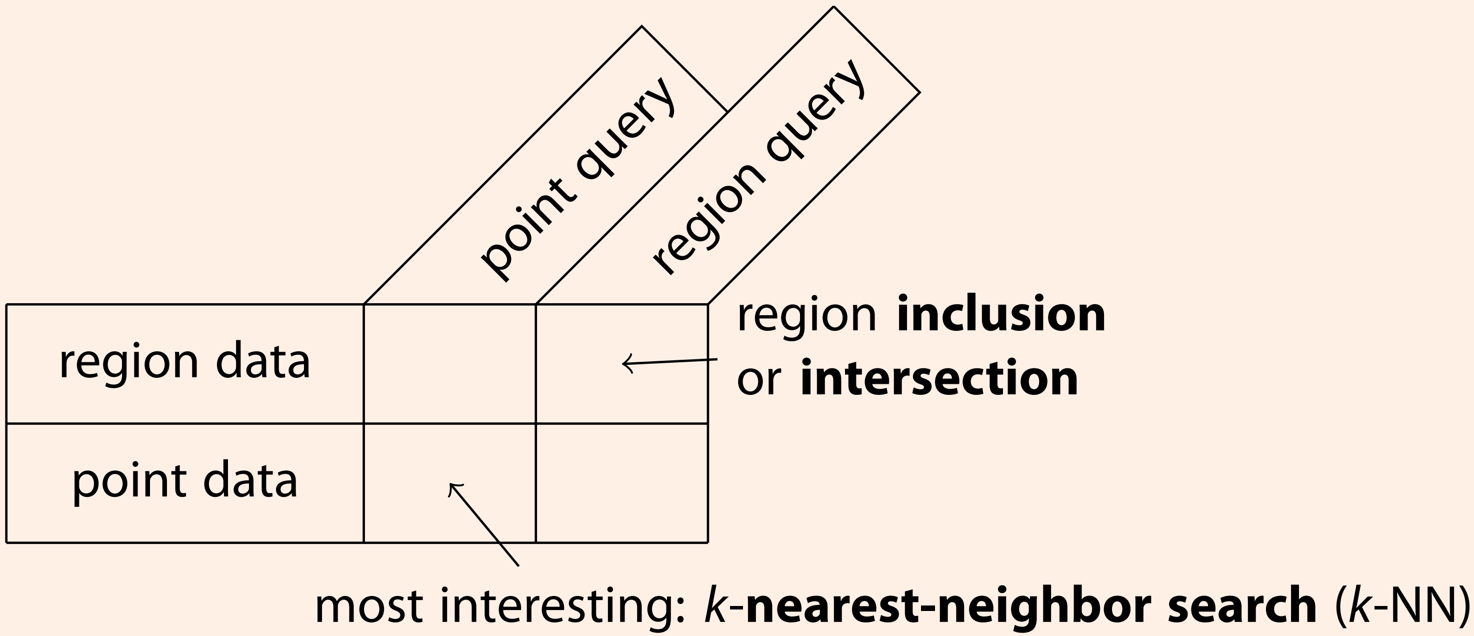
This query involves a **range predicate** in **two** dimensions.

Typical use cases for **multi-dimensional data** include

- **geographical data** (GIS: Geographical Information Systems),
- **multimedia retrieval** (*e.g.*, image or video search),
- **OLAP** (Online Analytical Processing),
- queries against **encoded data** (*e.g.*, XML)

. . . More Challenges. . .

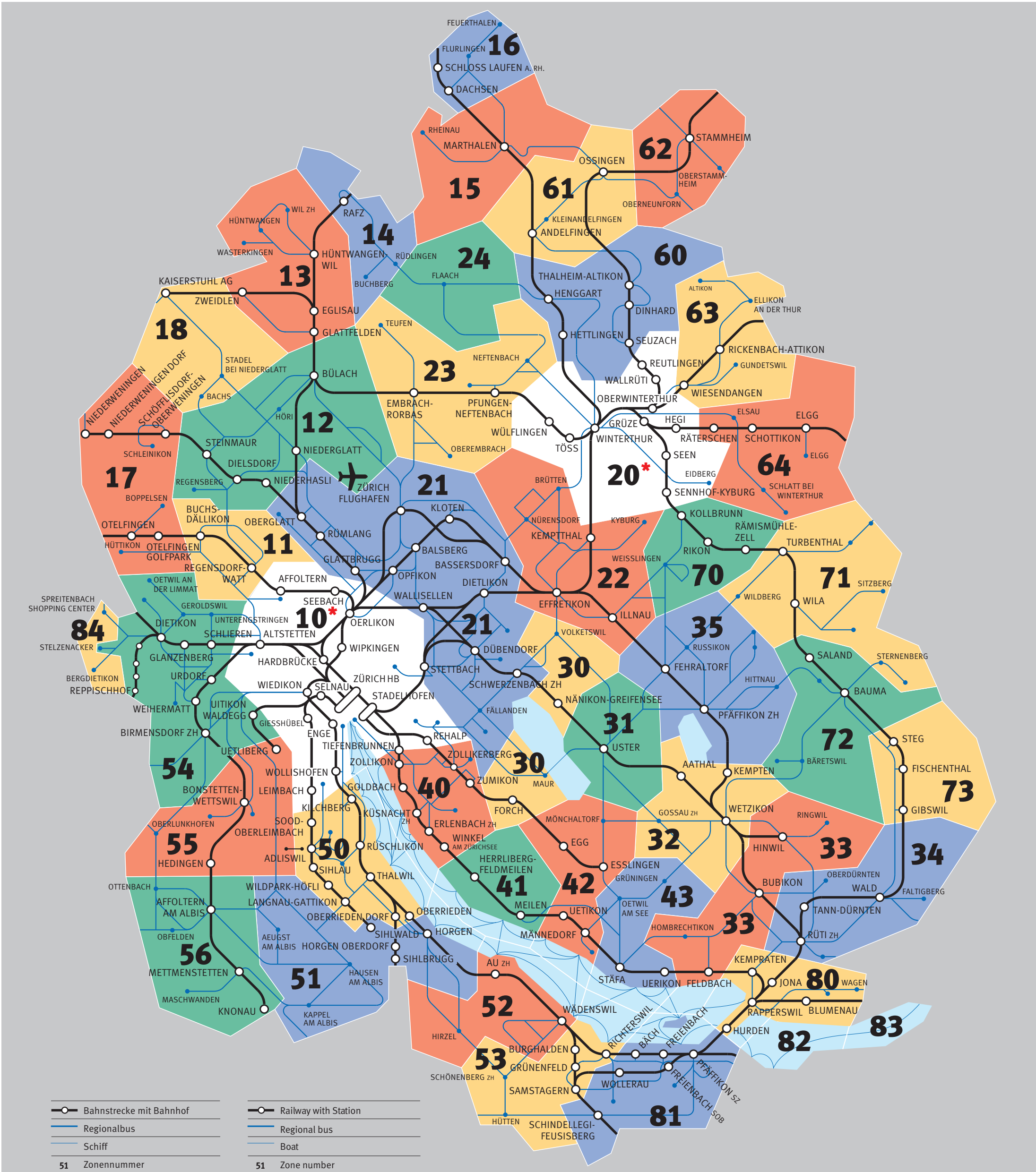
Queries and data can be points or regions



. . . and you can come up with many more meaningful types of queries over multi-dimensional data.

Note: All equality searches can be reduced to one-dimensional queries.

Points, Lines, and Regions

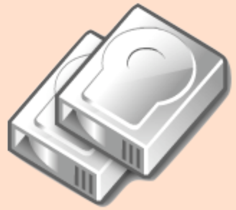


...More Solutions

Recent proposals for multi-dimension index structures

Quad Tree [Finkel 1974]	K-D-B-Tree [Robinson 1981]
R-tree [Guttman 1984]	Grid File [Nievergelt 1984]
R$^+$-tree [Sellis 1987]	LSD-tree [Henrich 1989]
R*-tree [Geckmann 1990]	hB-tree [Lomet 1990]
Vp-tree [Chiueh 1994]	TV-tree [Lin 1994]
UB-tree [Bayer 1996]	hB-$^{\Pi}$-tree [Evangelidis 1995]
SS-tree [White 1996]	X-tree [Berchtold 1996]
M-tree [Ciaccia 1996]	SR-tree [Katayama 1997]
Pyramid [Berchtold 1998]	Hybrid-tree [Chakrabarti 1999]
DABS-tree [Böhm 1999]	IQ-tree [Böhm 2000]
Slim-tree [Faloutsos 2000]	Landmark file [Böhm 2000]
P-Sphere-tree [Goldstein 2000]	A-tree [Sakurai 2000]

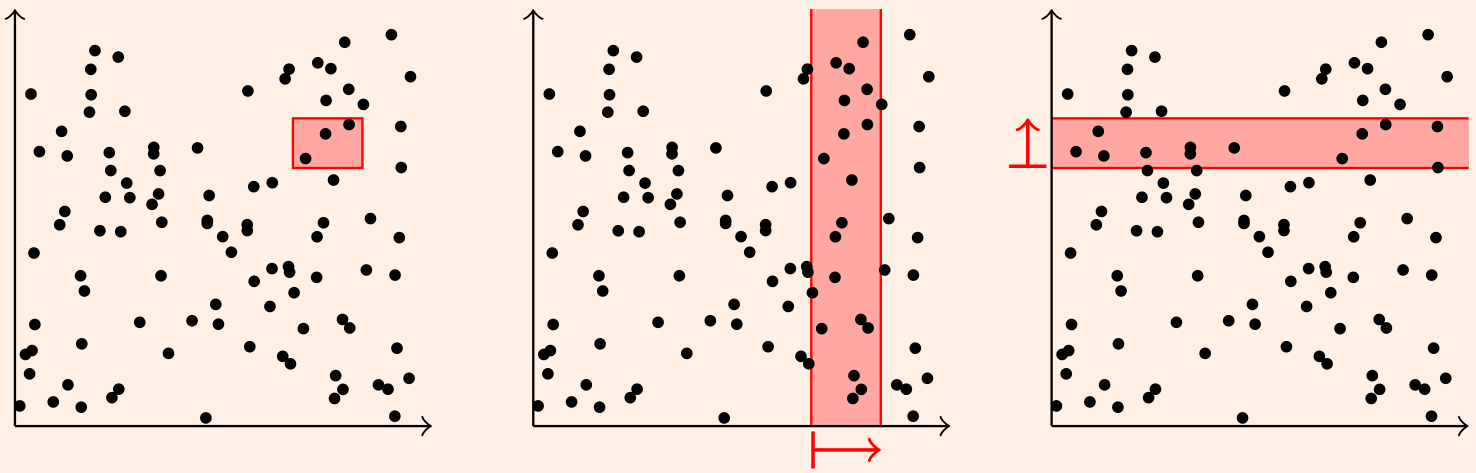
None of these provides a “fits all” solution.



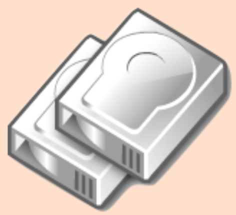
Can We Just Use a B$^+$-tree?

- Can two B$^+$-trees, one over `ZIPCODE` and over `REVENUE`, adequately support a two-dimensional range query?

Querying a rectangle



- Can only scan along **either** index at once, and both of them produce many **false hits**.
- If all you have are these two indices, you can do **index intersection:** perform both scans in separation to obtain the *rids* of candidate tuples. Then compute the intersection between the two *rid* lists (DB2: `IXAND`).



IBM DB2: Index Intersection

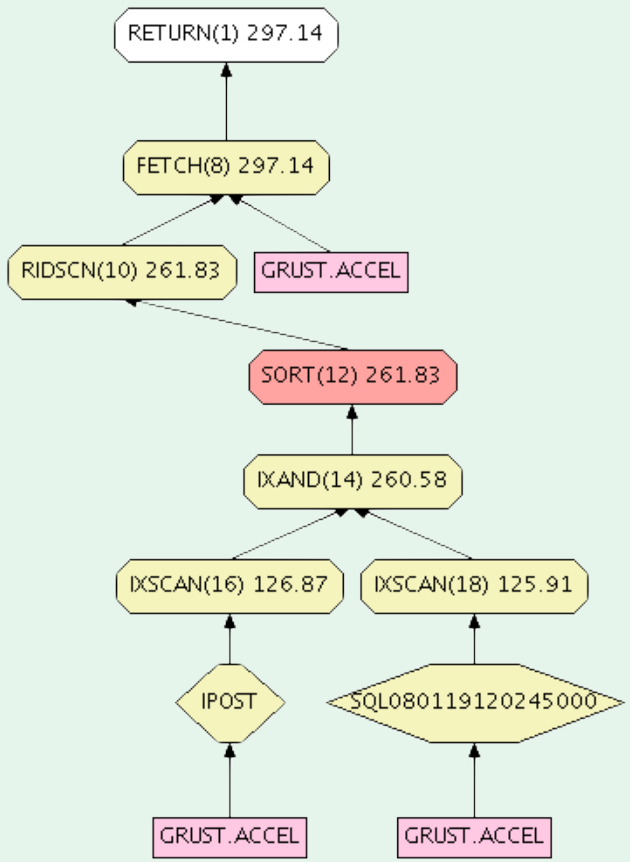
IBM DB2 uses index intersection (operator IXAND)

```sql
SELECT *
FROM   ACCEL
WHERE  PRE BETWEEN 0 AND 10000
AND    POST BETWEEN 10000 AND 20000
```

Relevant indexes defined over table ACCEL:

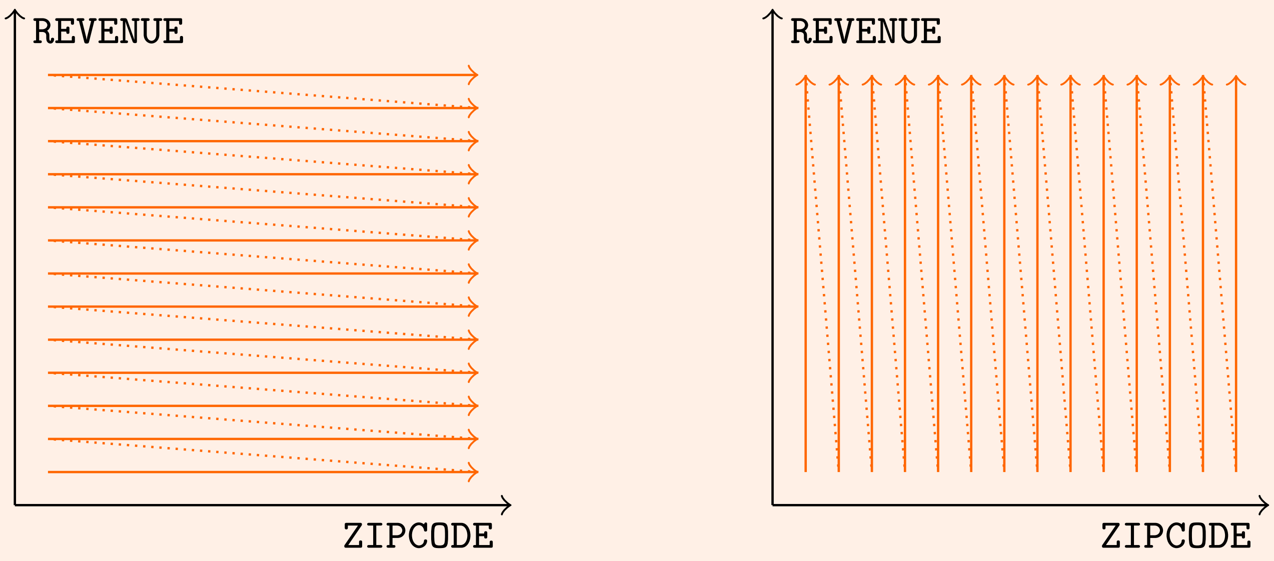
1. IPOST over column POST
2. SQL0801192024500 over column PRE (primary key)

Plan selected by the query optimizer



Can Composite Keys Help?

Indexes with composite keys



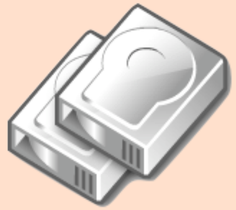
$\langle$REVENUE, ZIPCODE$\rangle$ index $\langle$ZIPCODE, REVENUE$\rangle$ index

- **Almost the same thing!**
  Indices over composite keys are **not symmetric**: The major attribute dominates the organization of the B$^+$-tree.
- **But**: Since the minor attribute is also stored in the index (part of the keys *k*), we may discard non-qualifying tuples **before** fetching them from the data pages.

Multi-Dimensional Indices

- B^+-trees can answer **one-dimensional** queries **only**.[1]
- We would like to have a **multi-dimensional index structure** that
  - is **symmetric** in its dimensions,
  - **clusters** data in a space-aware fashion,
  - is **dynamic** with respect to updates, and
  - provides good support for useful queries.
- We will start with data structures that have been designed for **in-memory** use, then tweak them into **disk-aware** database indices (*e.g.*, organize data in a page-wise fashion).

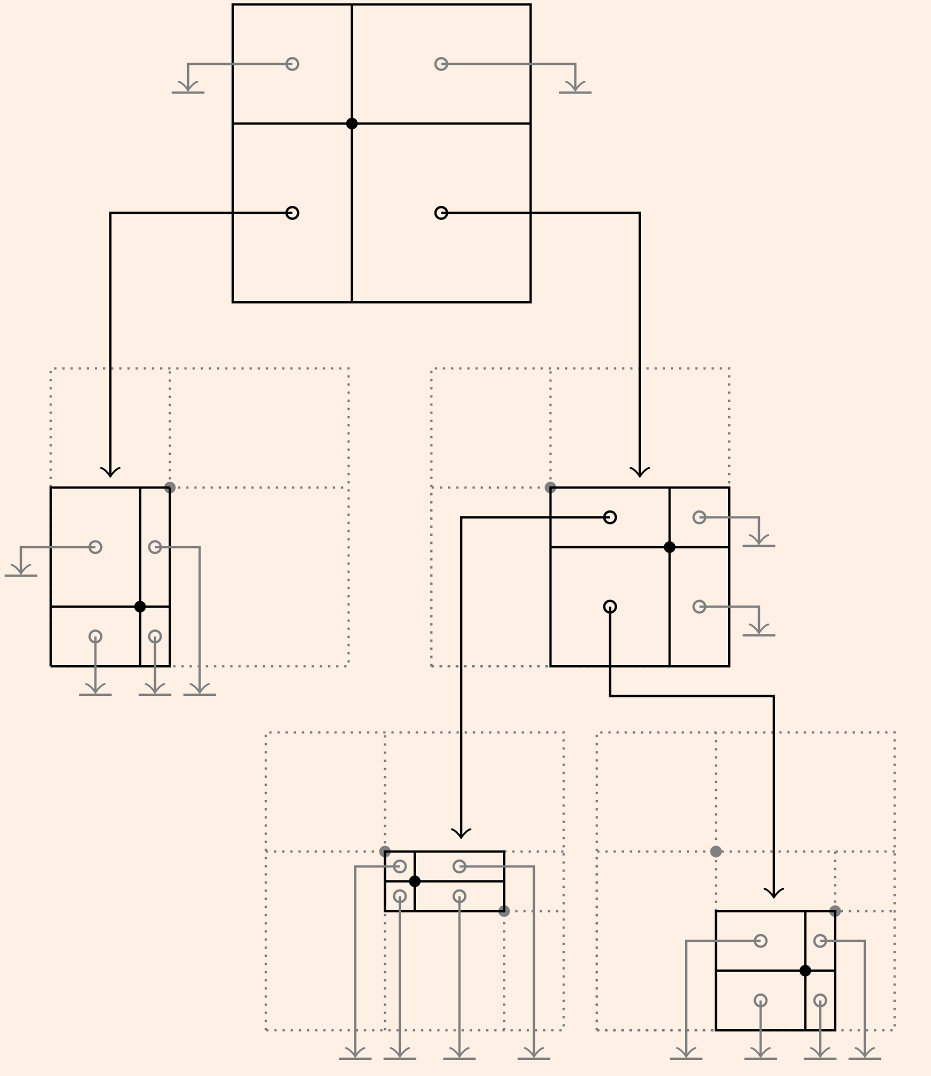
[1] Toward the end of this chapter, we will see UB-trees, a nifty trick that uses B^+-trees to support some multi-dimensional queries.



"Binary" Search Tree

In k dimensions, a **"binary tree"** becomes a 2^k**-ary tree**.

Point quad tree ($k = 2$)

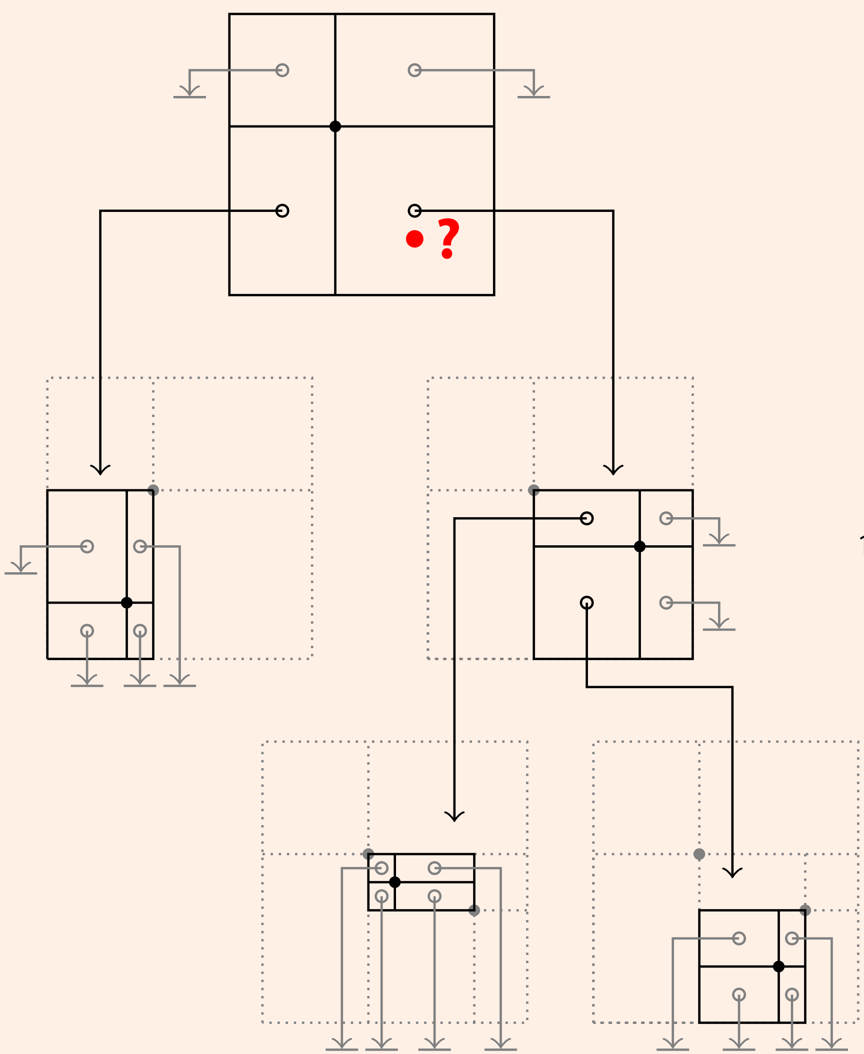


- Each data point **partitions** the data space into 2^k **disjoint regions**.
- In each node, a region points to another node (representing a refined partitioning) or to a special **null** value.
- This data structure is a **point quad tree**.

  ↗ Finkel and Bentley. Quad Trees: A Data Structure for Retrieval on Composite Keys. *Acta Informatica*, vol. 4, 1974.

Searching a Point Quad Tree

Point quad tree $(k = 2)$



Point quad tree search

Function: `p_search` $(q, node)$

if q matches data point in *node* **then**
  return data point;

else
  $P \leftarrow$ partition containing q;
  if P points to null **then**
    return not found;

  else
    $node' \leftarrow$ node pointed to by P;
    return `p_search` $(q, node')$;

Function: `pointsearch` (q)

return `p_search` $(q, root)$;

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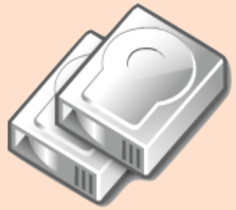

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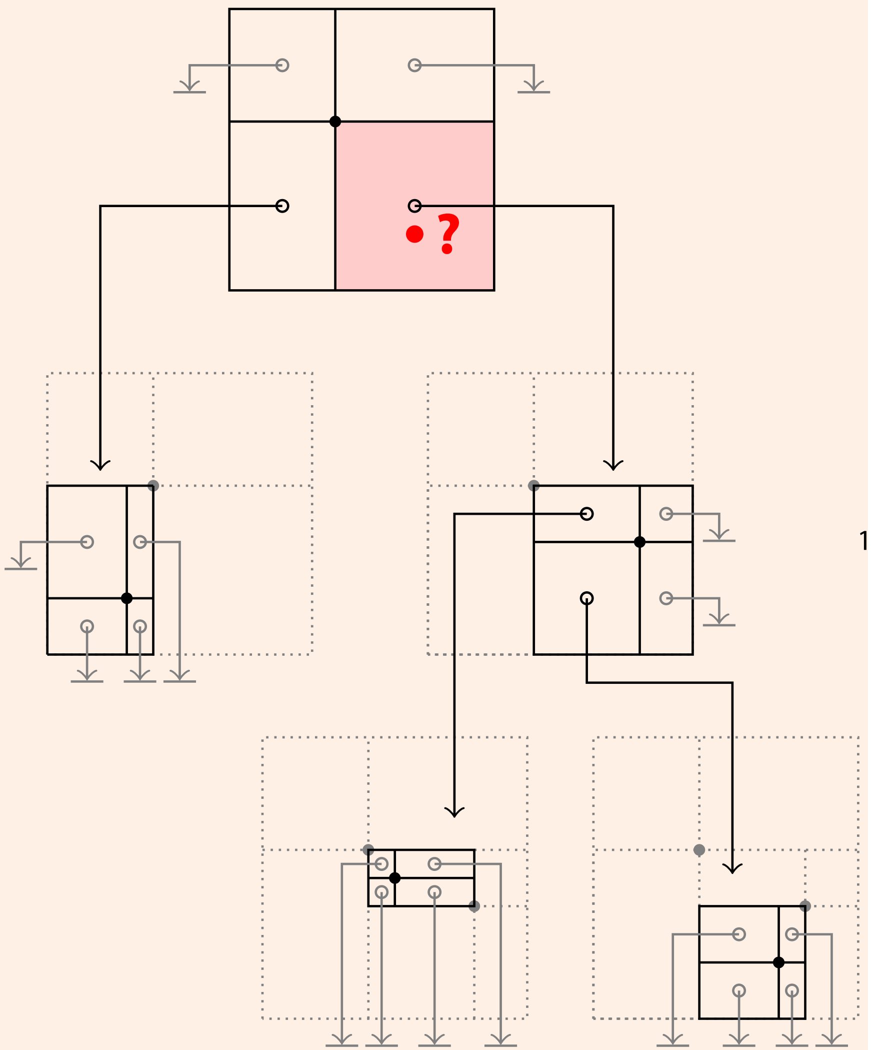

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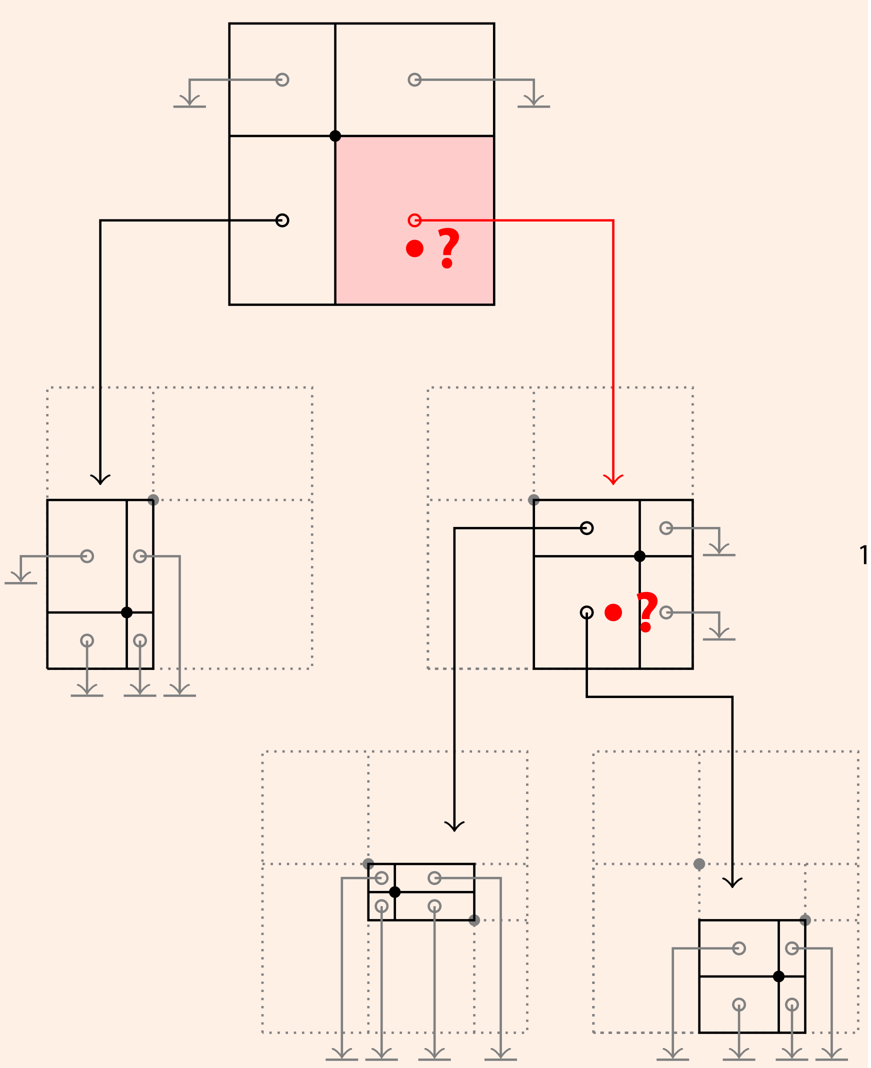

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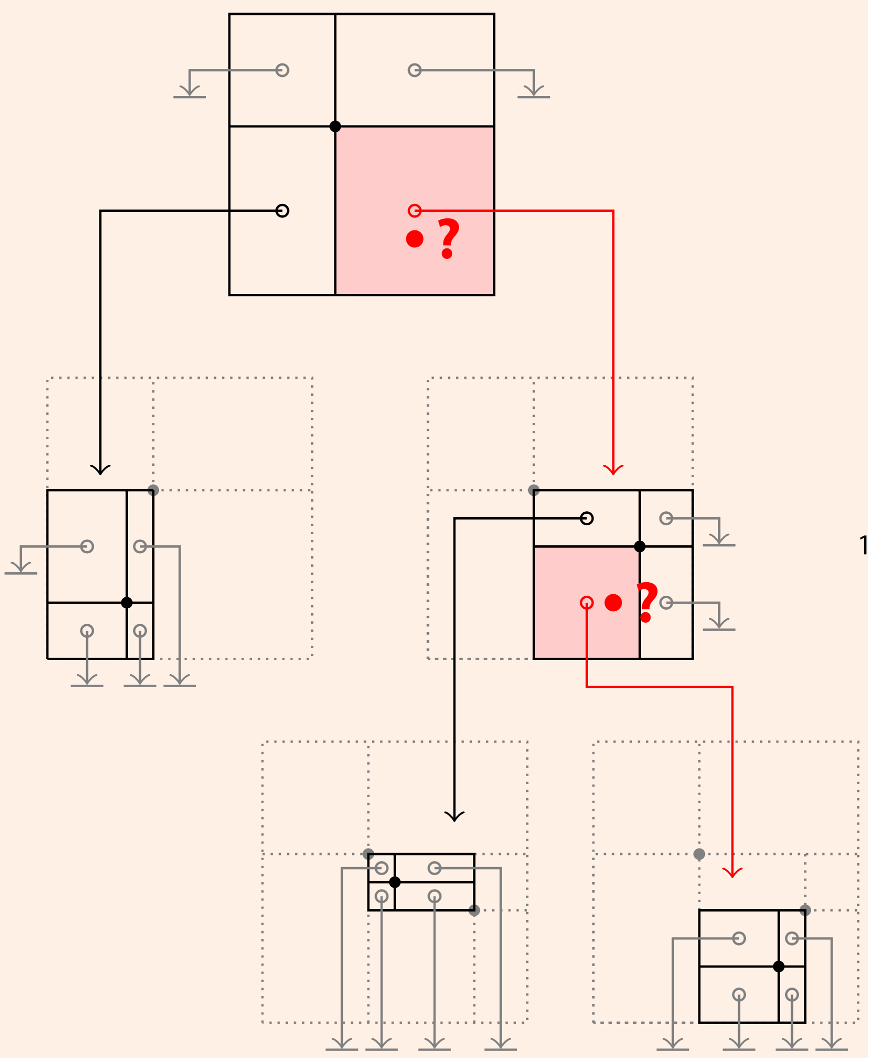

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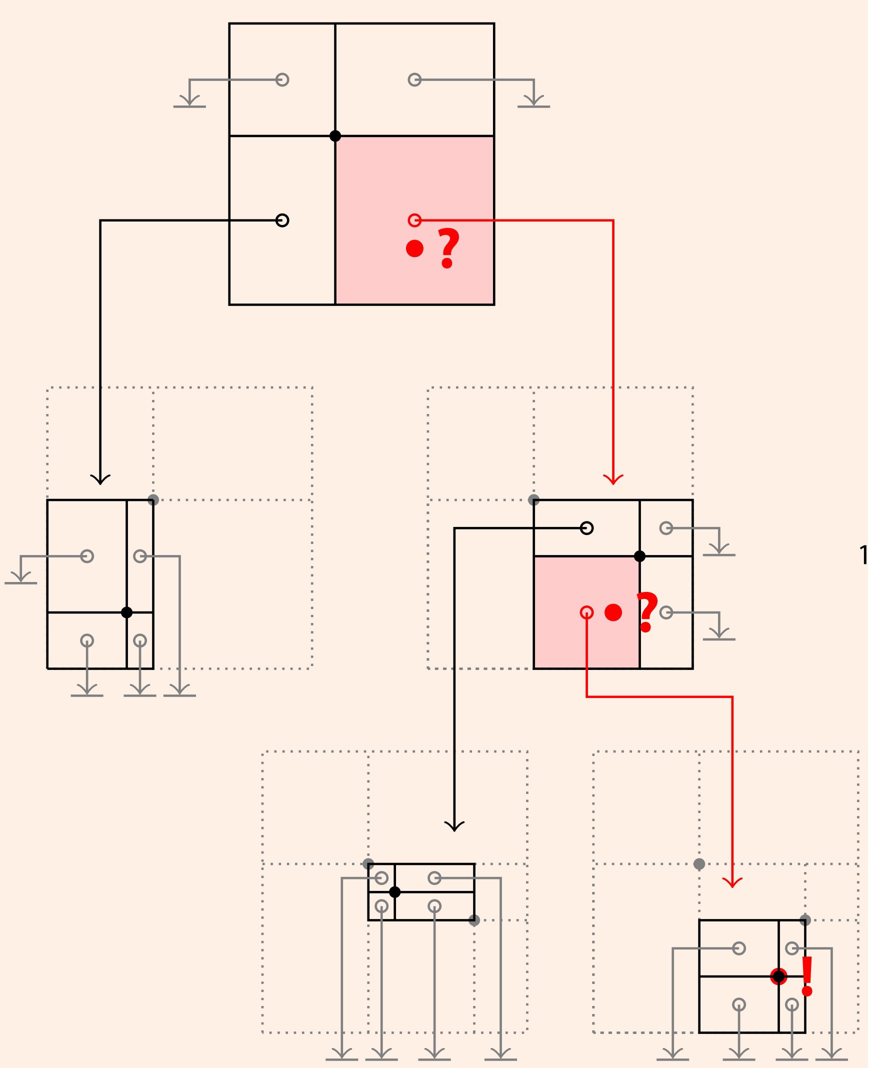

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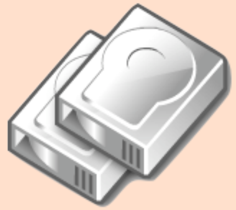


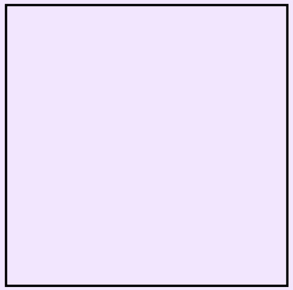
Inserting into a Point Quad Tree

Inserting a point q_{new} into a quad tree happens analogously to an insertion into a binary tree:

1. **Traverse** the tree just like during a search for q_{new} until you encounter a partition P with a **null** pointer.
2. Create a **new node** n' that spans the same area as P and is partitioned by q_{new}, with all partitions pointing to **null**.
3. Let P point to n'.

Note that this procedure does **not** keep the tree **balanced**.

Example (Insertions into an empty point quad tree)



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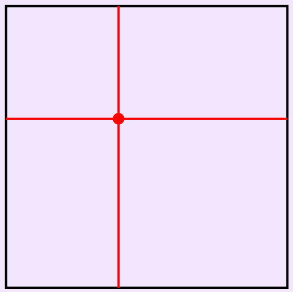

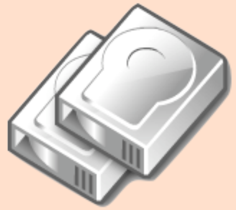

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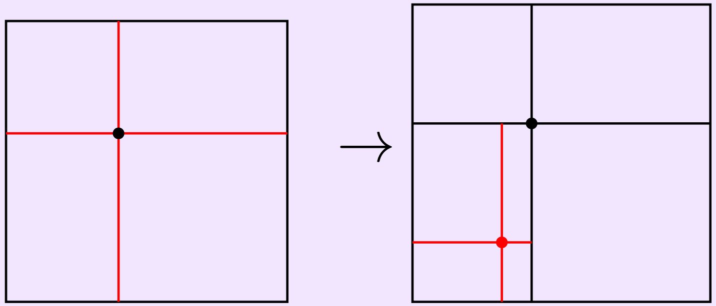

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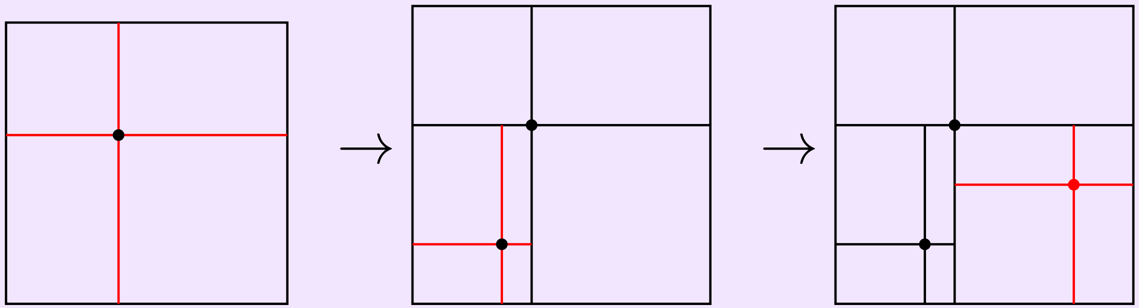

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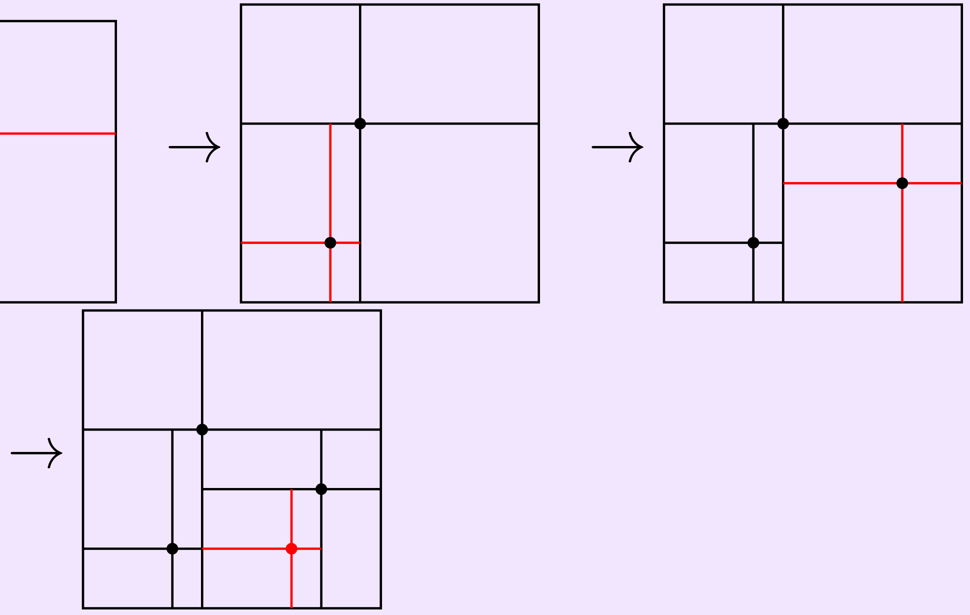

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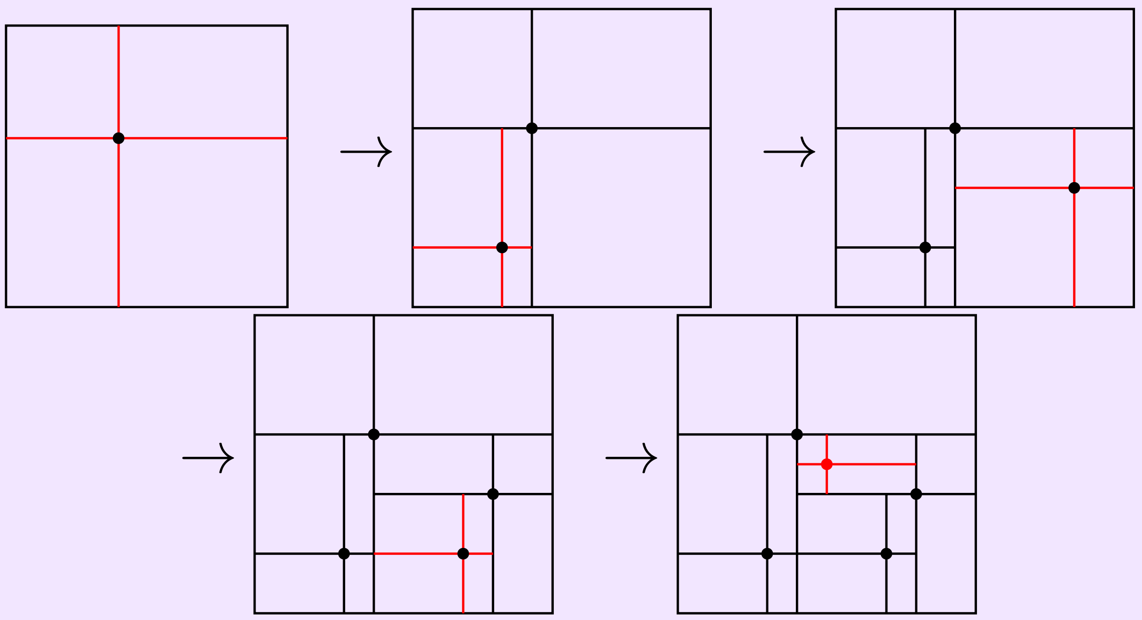
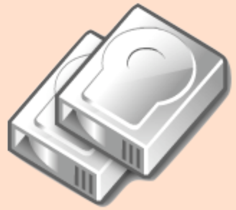




Range Queries

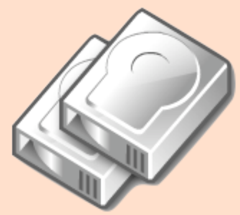
To evaluate a **range query**[2], we may need to follow **several** children of a quad tree node *node*:

Point quad tree range search

```
Function: r_search (Q, node)
if data point in node is in Q then
    append data point to result ;
foreach partition P in node that intersects with Q do
    node′ ← node pointed to by P ;
    r_search (Q, node′) ;
```

```
Function: regionsearch (Q)
return r_search (Q, root) ;
```

[2] We consider **rectangular** regions only; other shapes may be answered by querying for the **bounding rectangle** and post-processing the output.



Point Quad Trees—Discussion

Point quad trees

- ✔ are **symmetric** with respect to all dimensions and
- ✔ can answer **point queries** and **region queries**.

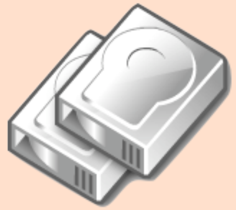
But

- ✗ the shape of a quad tree depends on the **insertion order** of its content, in the worst case **degenerates** into a **linked list**,
- ✗ **null** pointers are **space inefficient** (particularly for large *k*).

In addition,

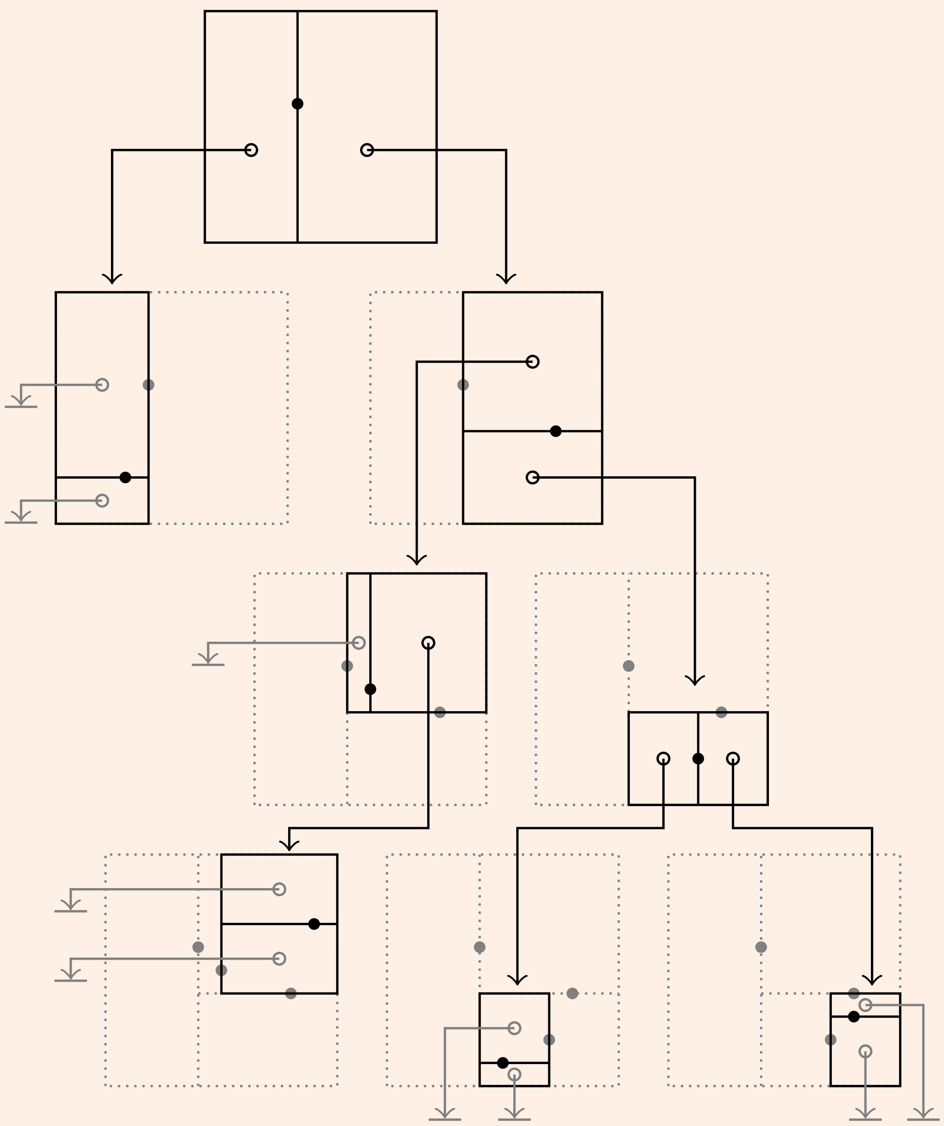
- they can only store **point data**.

Also remember that quad trees were designed for **main memory** operation.



k-d Trees

Sample *k*-d tree ($k = 2$)



- Index *k*-dimensional data, but keep the tree **binary**.
- For each **tree level** *l* use a different **discriminator dimension** d_l along which to **partition**.
  - Typically: **round robin**
- This is a ***k*-d tree**.

  ↗ Bentley. Multidimensional Binary Search Trees Used for Associative Searching. *Comm. ACM*, vol. 18, no. 9, Sept. 1975.

k-d Trees

k-d trees inherit the positive properties of the point quad trees, but improve on **space efficiency**.

For a given point set, we can also construct a **balanced** k-d tree:[3]

k-d tree construction (Bentley's *OPTIMIZE* algorithm)

Function: `kdtree` $(\mathit{pointset}, \mathit{level})$

if *pointset* is empty **then**

  return null ;

else

  $p \leftarrow$ median from *pointset* (along d_{level}) ;

  $\mathit{points}_{\mathrm{left}} \leftarrow \{v \in \mathit{pointset} \text{ where } v_{d_{\mathit{level}}} < p_{d_{\mathit{level}}}\}$;

  $\mathit{points}_{\mathrm{right}} \leftarrow \{v \in \mathit{pointset} \text{ where } v_{d_{\mathit{level}}} \geq p_{d_{\mathit{level}}}\}$;

  $n \leftarrow$ new k-d tree node, with data point p ;

  $n.\mathit{left} \leftarrow$ `kdtree` $(\mathit{points}_{\mathrm{left}}, \mathit{level} + 1)$;

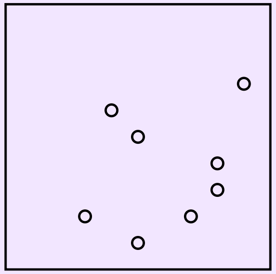
  $n.\mathit{right} \leftarrow$ `kdtree` $(\mathit{points}_{\mathrm{right}}, \mathit{level} + 1)$;

  return n ;

[3] v_i: coordinate i of point v.

Balanced k-d Tree Construction

Example (Step-by-step balanced k-d tree construction)



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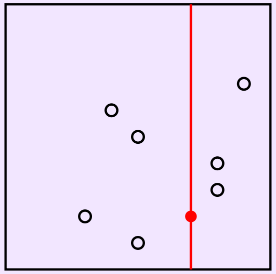

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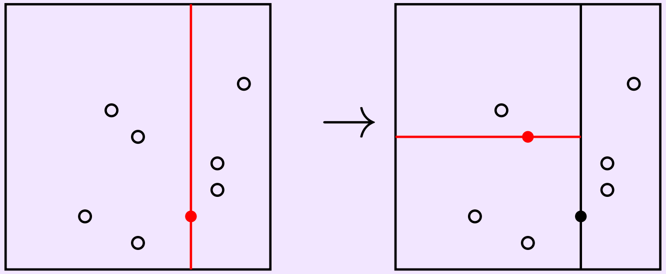

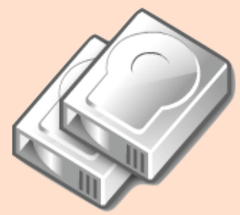

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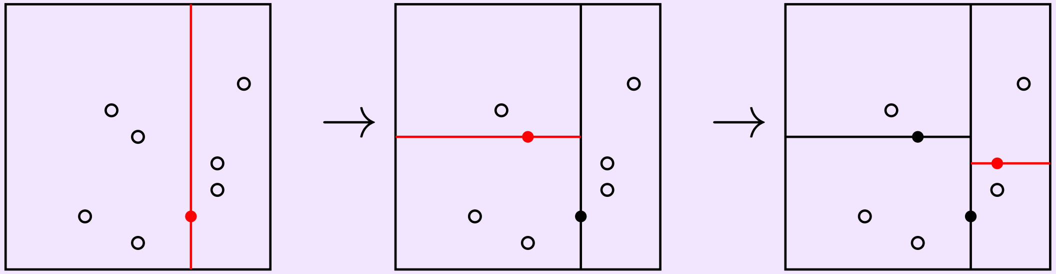

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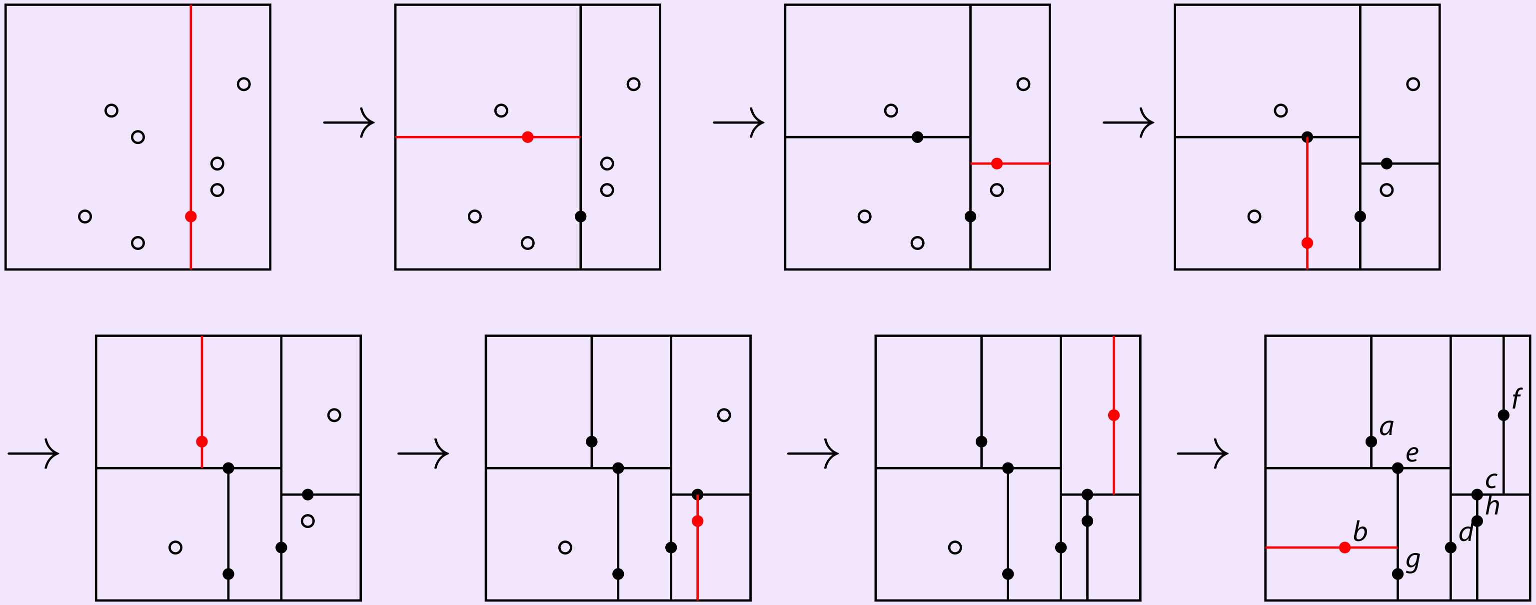

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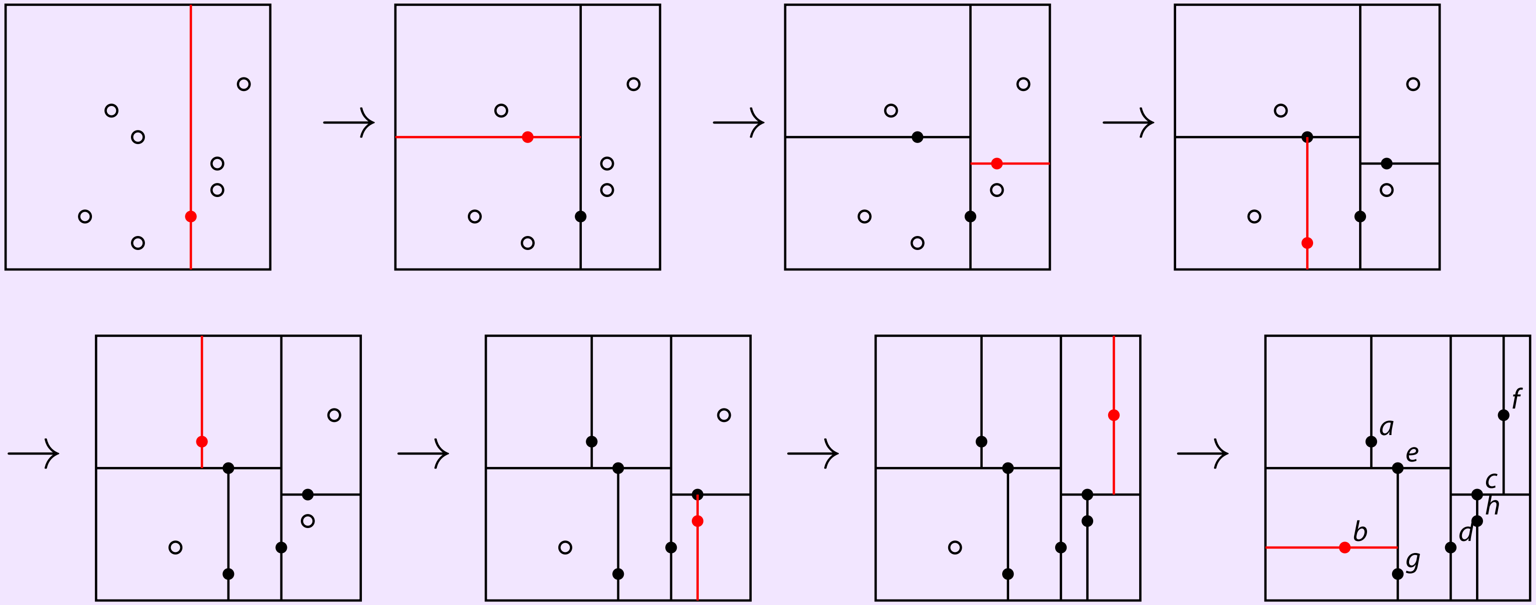
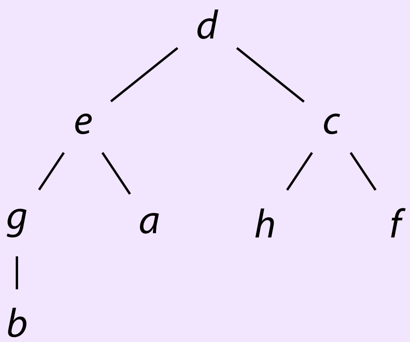


Resulting tree shape:



K-D-B-Trees

k-d trees improve on some of the deficiencies of point quad trees:

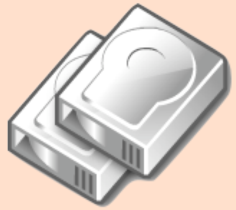
✔ We can **balance** a *k*-d tree by **re-building** it.
(For a limited number of points and in-memory processing, this may be sufficient.)

✔ We're no longer wasting big amounts of **space**.

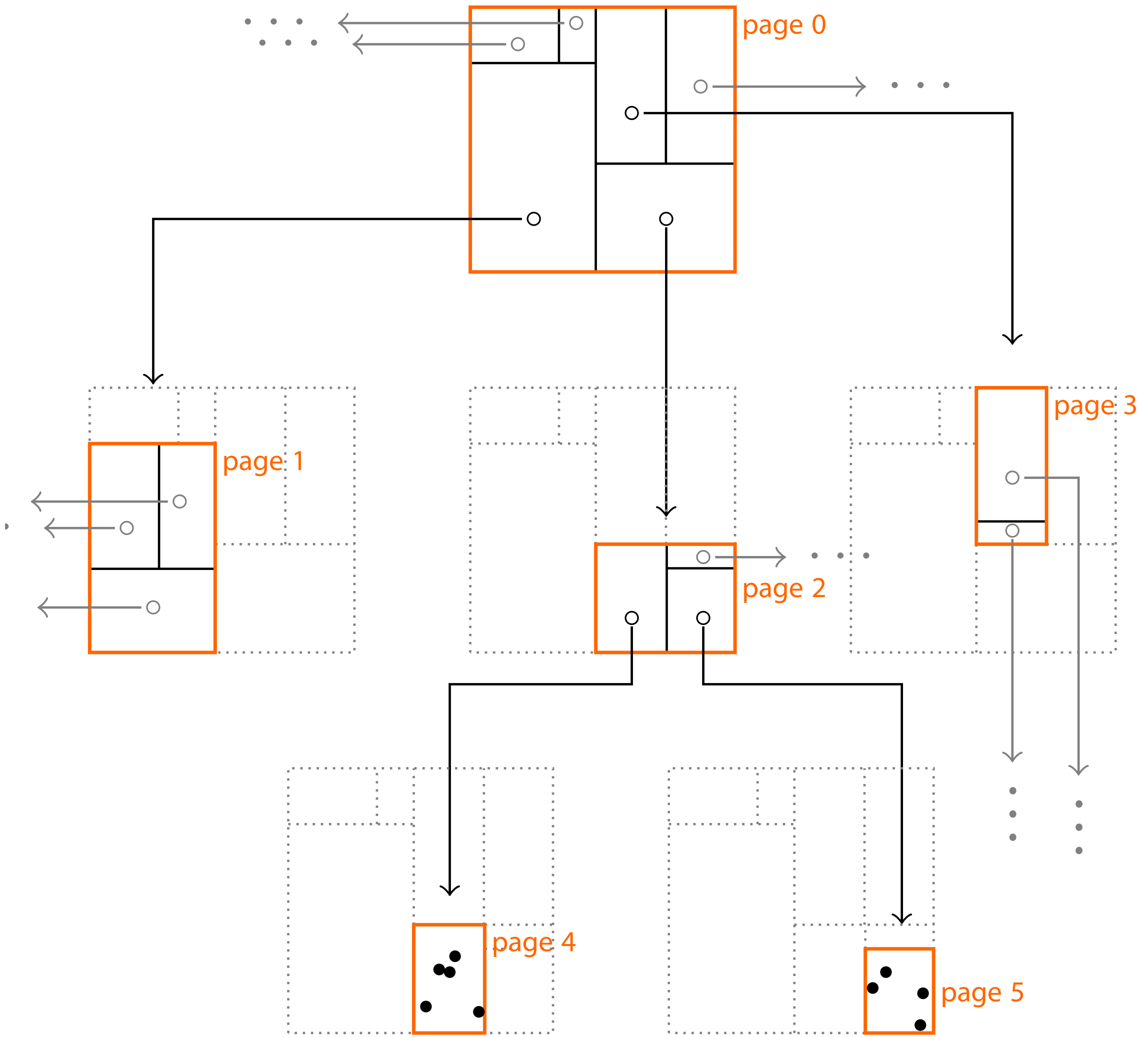
It's time to bring *k*-d trees to the disk. The **K-D-B-Tree**

- uses **pages** as an organizational unit
(*e.g.*, each node in the K-D-B-tree fills a page) and
- uses a ***k*-d tree-like layout** to organize each page.

↗ John T. Robinson. The K-D-B-Tree: A Search Structure for Large Multidimensional Dynamic Indexes. *SIGMOD 1981*.



K-D-B-Tree Idea



Region pages:

- contain entries $\langle \mathit{region}, \mathit{pageID} \rangle$
- no **null** pointers
- form a **balanced** tree
- all regions **disjoint** and **rectangular**

Point pages:

- contain entries $\langle \mathit{point}, \mathit{rid} \rangle$
- $\rightsquigarrow$ B^+-tree leaf nodes

K-D-B-Tree Operations

- **Searching** a K-D-B-Tree works straightforwardly:
  - Within each page determine all regions R_i that contain the query point q (intersect with the query region Q).
  - For each of the R_i, consult the page it points to and recurse.
  - On point pages, fetch and return the corresponding record for each matching data point p_i.

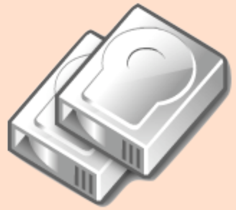
- When **inserting** data, we keep the K-D-B-Tree **balanced**, much like we did in the **B$^+$-tree**:
  - Simply insert a $\langle region, pageID\rangle$ ($\langle point, rid\rangle$) entry into a region page (point page) if there's **sufficient space**.
  - **Otherwise**, **split** the page.

K-D-B-Tree: Splitting a Point Page

Splitting a point page *p*

1. **Choose a dimension** i and an i-coordinate x_i along which to split, such that the split will result in two pages that are not overfull.
2. **Move** data points p with $p_i < x_i$ and $p_i \geqslant x_i$ to new pages p_{left} and p_{right} (respectively).
3. Replace $\langle \textit{region}, p \rangle$ in the **parent** of p with $\langle \textit{left region}, p_{\text{left}} \rangle$ $\langle \textit{right region}, p_{\text{right}} \rangle$.

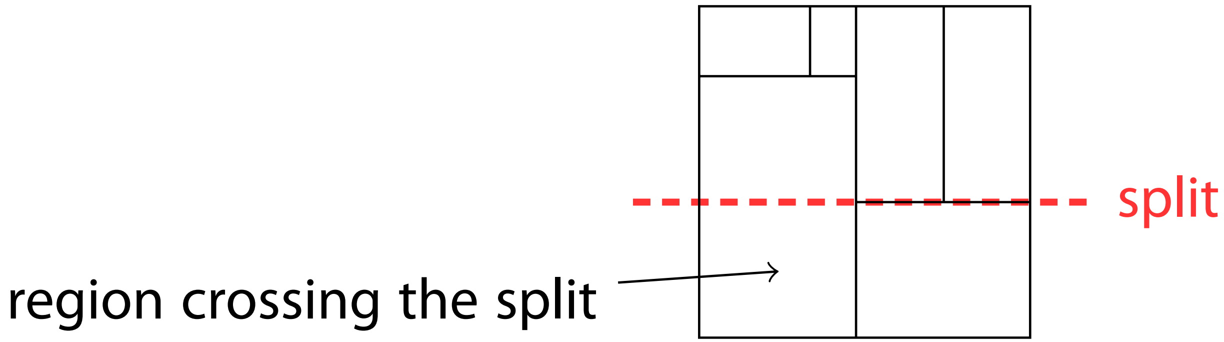
Step 3 may cause an **overflow** in p's parent and, hence, lead to a **split** of a **region page**.



K-D-B-Tree: Splitting a Region Page

- Splitting a **point page** and moving its data **points** to the resulting pages is straightforward.
- In case of a **region page split**, by contrast, some **regions** may intersect with **both** sides of the split.

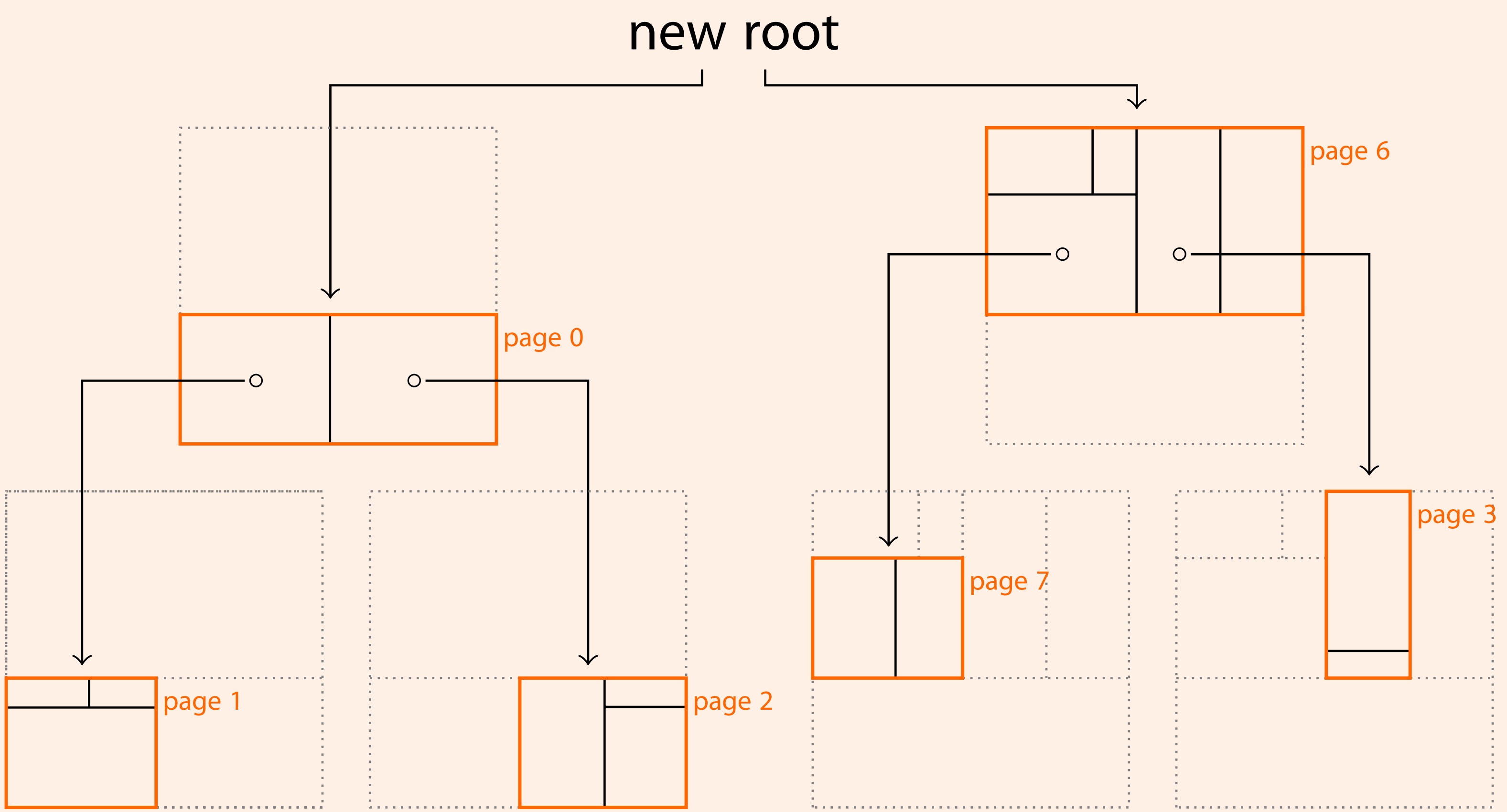
  Consider, *e.g.*, page 0 on slide 19:



- Such regions need to be **split**, too.
- This can cause a **recursive** splitting **downward** (!) the tree.

Example: Page 0 Split in K-D-B-Tree of slide 19

Split of region page 0



- Root page 0 ⇒ pages 0 and 6 (⇝ creation of new root).
- Region page 1 ⇒ pages 1 and 7 (point pages not shown).

K-D-B-Trees: Discussion

K-D-B-Trees

- ✔ are **symmetric** with respect to all dimensions,
- ✔ **cluster** data in a space-aware and page-oriented fashion,
- ✔ are **dynamic** with respect to updates, and
- ✔ can answer **point queries** and **region queries**.

However,

- we still don't have support for **region data** and
- K-D-B-Trees (like *k*-d trees) will not handle **deletes** dynamically.

This is because we always partitioned the data space such that

- every region is **rectangular** and
- regions never **intersect**.

R-trees do not have the disjointness requirement:

- R-tree inner or leaf nodes contain $\langle region, pageID\rangle$ or $\langle region, rid\rangle$ entries (respectively). *region* is the **minimum bounding rectangle** that spans all data items reachable by the respective pointer.
- Every node contains between d and $2d$ entries ($\rightsquigarrow$ B^+-tree).[4]
- **Insertion** and **deletion** algorithms keep an R-tree **balanced** at all times.

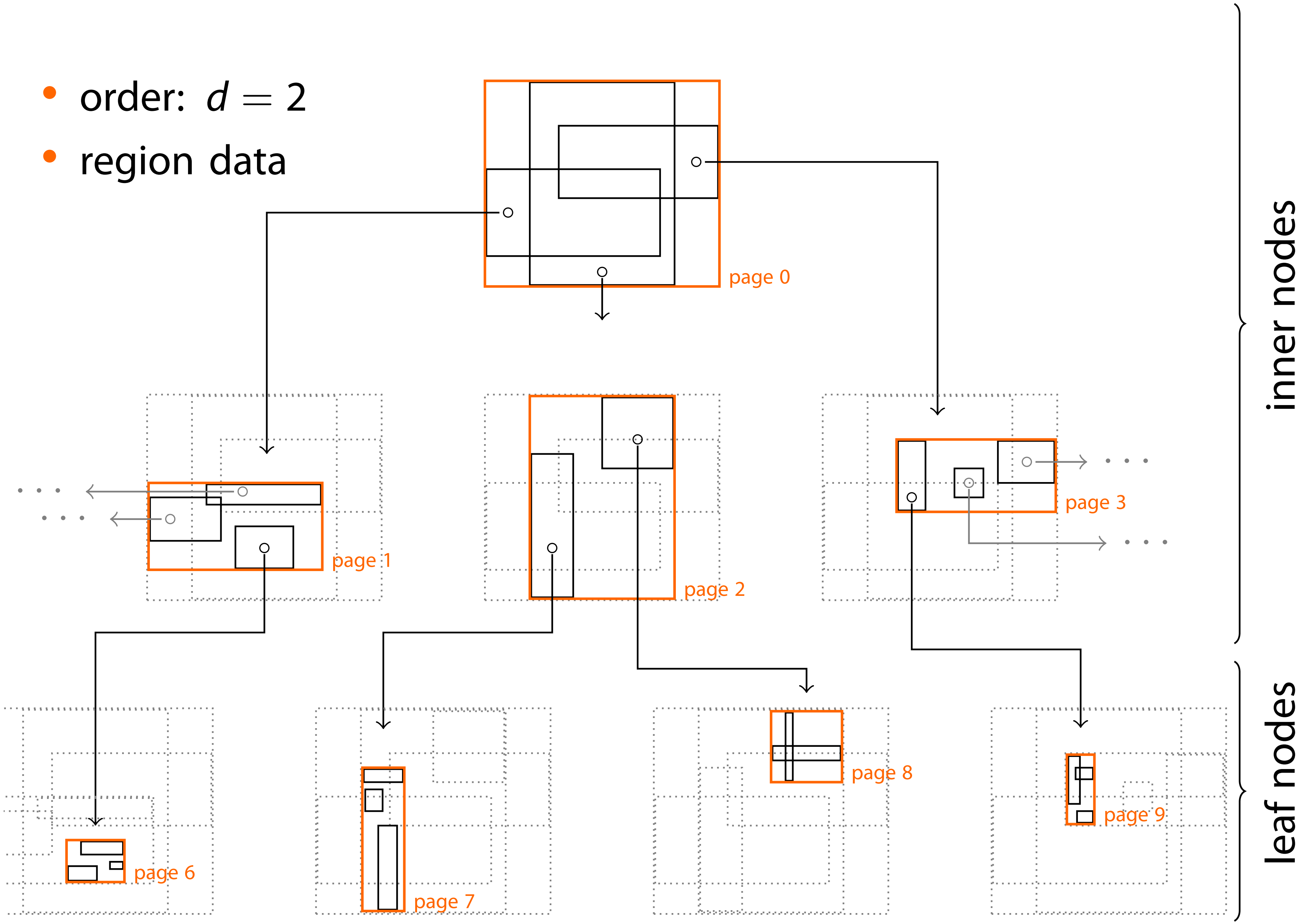
R-trees allow the storage of **point and region data**.

↗ Antonin Guttman. R-Trees: A Dynamic Index Structure for Spatial Searching. *SIGMOD 1984*.

[4] except the root node

R-Tree: Example

- order: $d = 2$
- region data

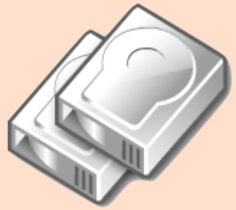


R-Tree: Searching and Inserting

While **searching** an R-tree, we may have to descend into more than one child node for point **and** region queries (↗ range search in point quad trees, slide 13).

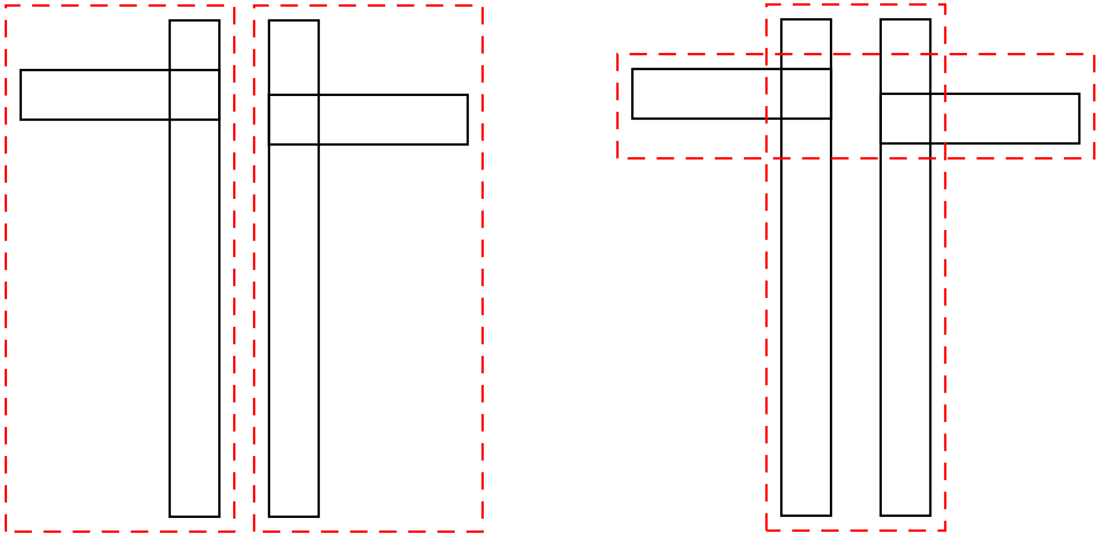
Inserting into an R-tree (cf. B$^+$-tree insertion)

1. **Choose** a leaf node n to insert the new entry.
   - Try to minimize the necessary region enlargement(s).
2. If n is **full**, **split** it (resulting in n and n') and distribute old and new entries evenly across n and n'.
   - Splits may propagate bottom-up and eventually reach the root (↗ B$^+$-tree).
3. After the insertion, some regions in the ancestor nodes of n may need to be adjusted to cover the new entry.



Splitting an R-Tree Node

To **split** an R-tree node, we generally have more than one alternative:



Heuristic: Minimize the totally covered area. But:

- **Exhaustive** search for the best split is infeasible.
- Guttman proposes two ways to **approximate** the search.
- Follow-up papers (*e.g.*, the R*-tree) aim at improving the quality of node splits.

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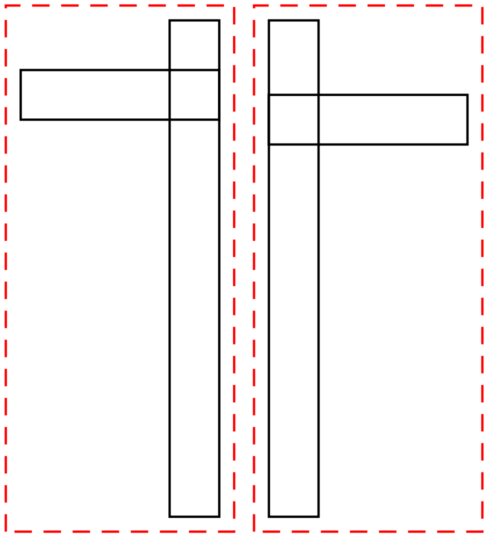
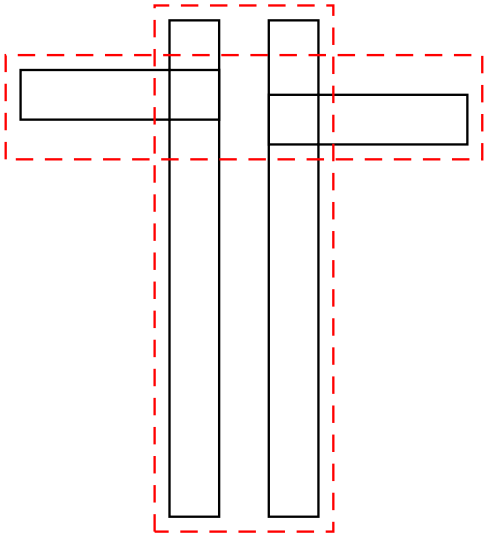


“bad” split



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Heuristic: Minimize the totally covered area. But:

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R-Tree: Deletes

All R-tree invariants (see 25) are maintained during **deletions**.

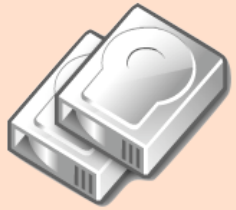
1. If an R-tree node *n* **underflows** (*i.e.*, less than *d* entries are left after a deletion), the whole node is **deleted**.
2. Then, all entries that existed in *n* are **re-inserted** into the R-tree.

Note that Step 1 may lead to a recursive deletion of *n*'s parent.

- Deletion, therefore, is a rather **expensive** task in an R-tree.

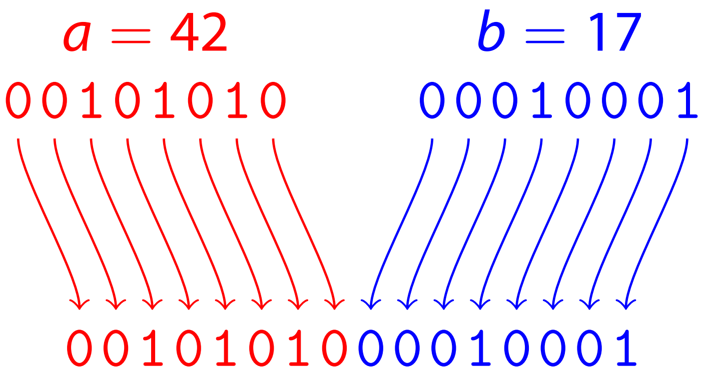
Spatial indexing in mainstream database implementations

- Indexing in commercial database systems is typically based on **R-trees**.
- Yet, only few systems implement them out of the box (*e.g.*, PostgreSQL). Most require the licensing/use of specific extensions.

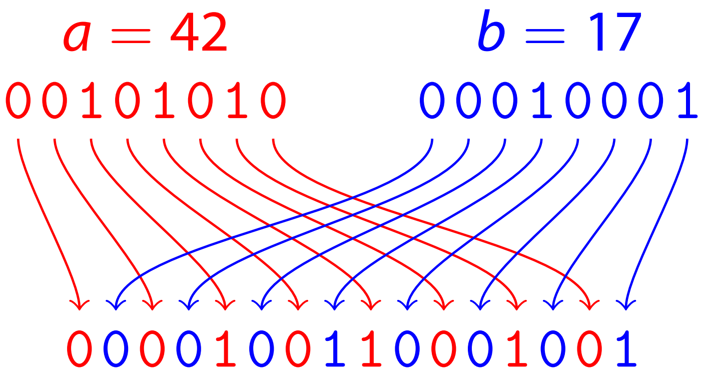


Bit Interleaving

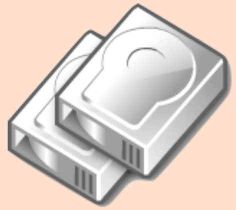
- We saw earlier that a B$^+$-tree over **concatenated** fields $\langle a, b \rangle$ does not help our case, because of the **asymmetry** between the role of a and b in the index key.
- What happens if we **interleave** the bits of a and b (hence, make the B$^+$-tree "more symmetric")?



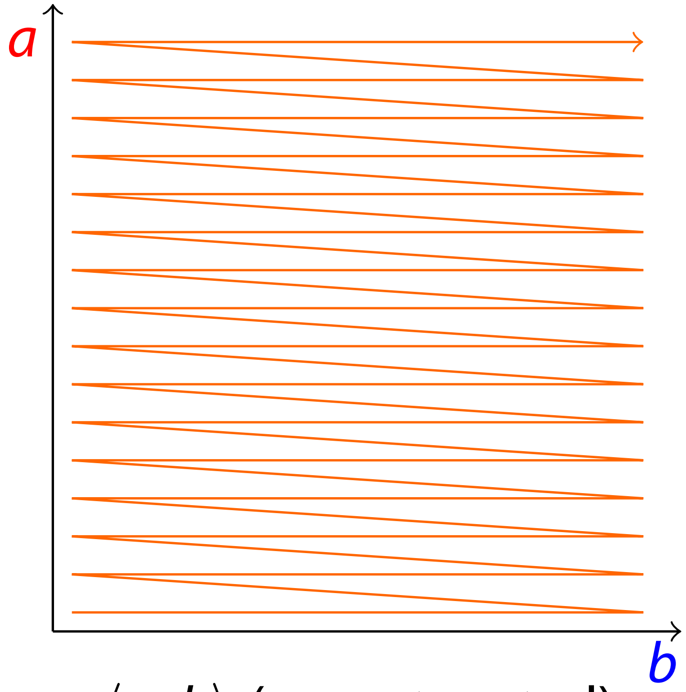
$\langle a, b \rangle$ (concatenation)



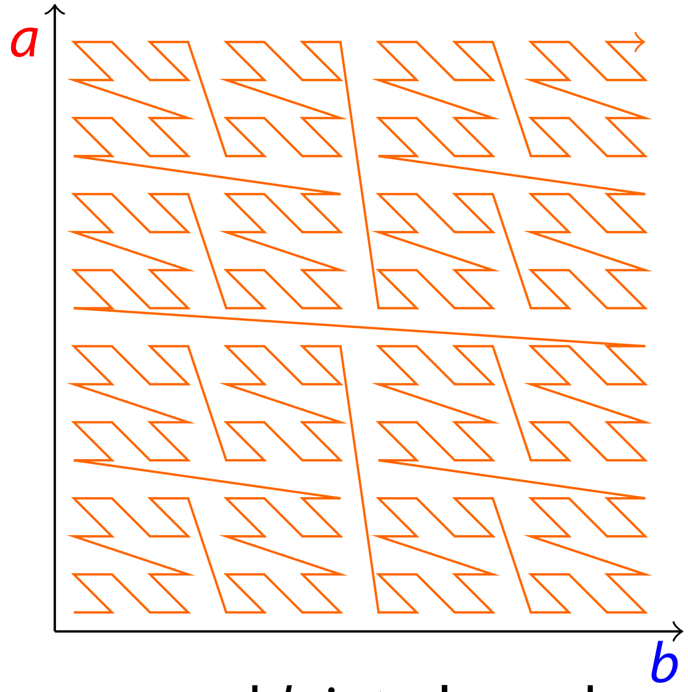
a and b interleaved



Z-Ordering



$\langle a, b \rangle$ (concatenated)



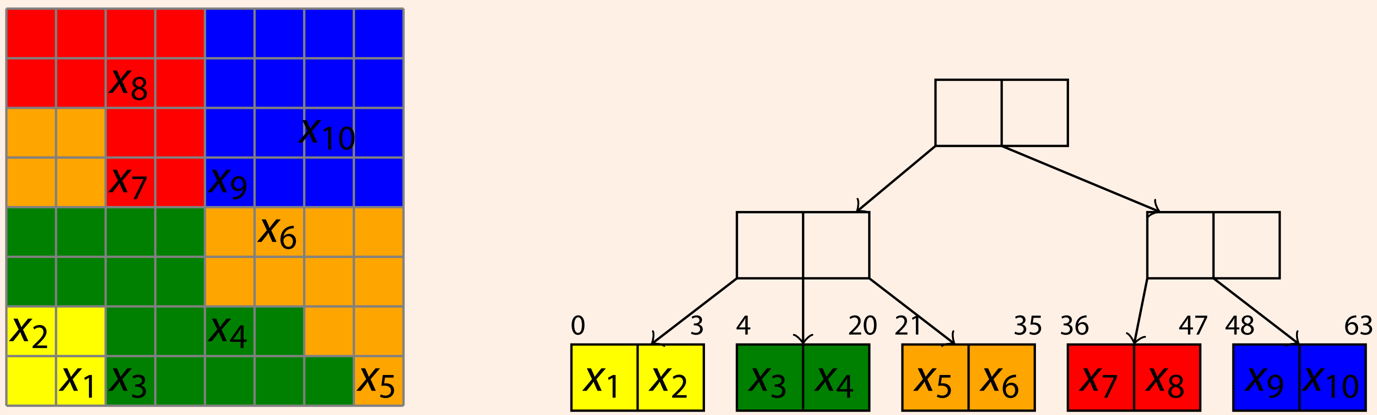
a and b interleaved

- Both approaches **linearize** all coordinates in the value space according to some **order**. ↗ see also slide 8
- Bit interleaving leads to what is called the **Z-order**.
- The Z-order (largely) preserves spatial **clustering**.

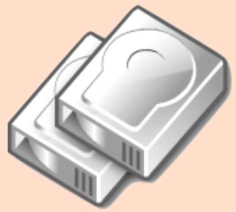
UB-Tree: B$^+$-trees Over Z-Codes

- Use a **B$^+$-tree** to index Z-codes of multi-dimensional data.
- Each leaf in the B$^+$-tree describes an **interval** in the **Z-space**.
- Each interval in the Z-space describes a **region** in the multi-dimensional data space:

UB-Tree: B$^+$-tree over Z-codes

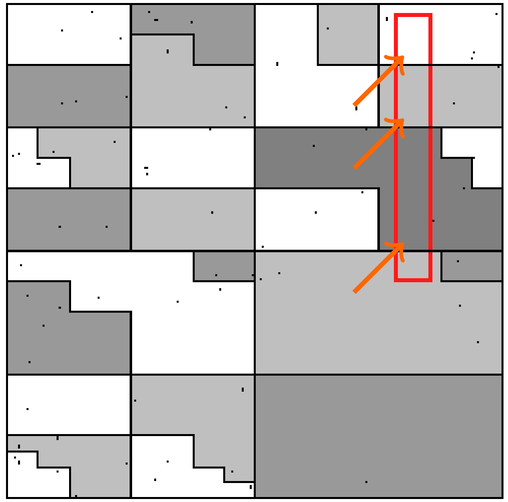


- To retrieve all data points in a query region Q, try to touch only those leave pages that **intersect** with Q.



UB-Tree: Range Queries

After each page processed, perform an **index re-scan** ($\nearrow$) to fetch the next leaf page that intersects with Q.



Example by Volker Markl and Rudolf Bayer, taken from http://mistral.in.tum.de/

UB-tree range search (Function `ub_range(Q)`)

$cur \leftarrow z(Q_{\text{bottom,left}})$;
while true **do**
  // search B$^+$-tree page containing cur ($\nearrow$ slide 0.0)
  $page \leftarrow$ `search` (cur);
  foreach data point p on $page$ **do**
    if p is in Q **then**
      append p to result ;
  if region in $page$ reaches beyond $Q_{\text{top,right}}$ **then**
    break ;
  // compute next Z-address using Q and data on current page
  $cur \leftarrow$ `get_next_z_address` (Q, $page$);

UB-Trees: Discussion

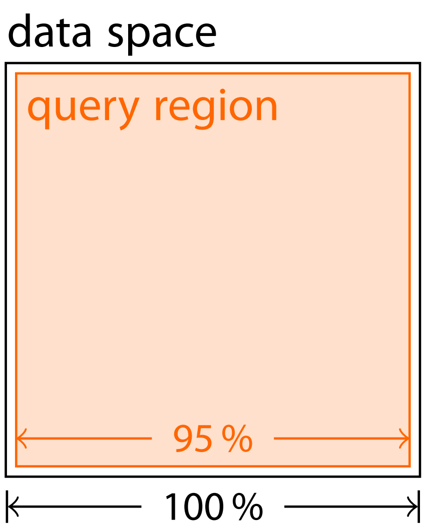
- Routine `get_next_z_address()` (return next Z-address lying in the query region) is non-trivial and depends on the shape of the Z-region.
- UB-trees are **fully dynamic**, a property inherited from the underlying B^+-trees.
- The use of other **space-filling curves** to linearize the data space is discussed in the literature, too. *E.g.*, **Hilbert curves**.

UB-trees have been commercialized in the Transbase® database system.

Spaces with High Dimensionality

For large k, all techniques that have been discussed become **ineffective**:

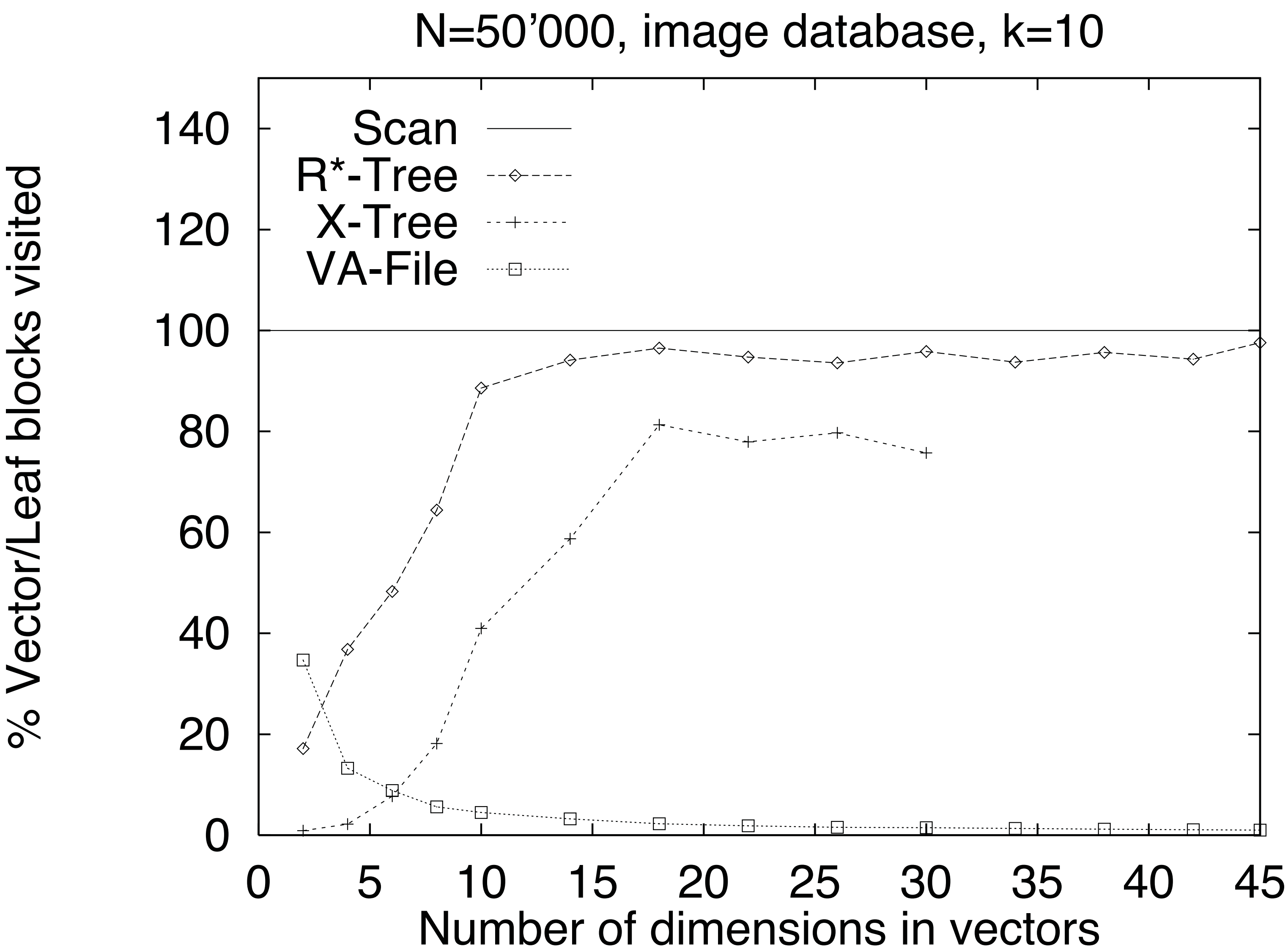
- For example, for $k = 100$, we get $2^{100} \approx 10^{30}$ partitions per node in a **point quad tree**. Even with billions of data points, **almost all** of these are empty.
- Consider a **really big** search region, cube-sized covering 95 % of the range along **each** dimension:



For $k = 100$, the probability of a point being in this region is still only $0.95^{100} \approx 0.59\,\%$.

- We experience the **curse of dimensionality** here.

Page Selectivty for *k*-NN Search



Data: Stephen Bloch. What's Wrong with High-Dimensionality Search. *VLDB 2008*.

Wrap-Up

Point Quad Tree

k-dimensional analogy to binary trees; main memory only.

k-d Tree, K-D-B-Tree

k-d tree: partition space one dimension at a time (round-robin); K-D-B-Tree: B^+-tree-like organization with pages as nodes, nodes use a *k*-d-like structure internally.

R-Tree

regions within a node may overlap; fully dynamic; for point and region data.

UB-Tree

use space-filling curve (Z-order) to linearize *k*-dimensional data, then use B^+-tree.

Curse Of Dimensionality

most indexing structures become ineffective for large *k*; fall back to sequential scanning and approximation/compression.