Part VI

Valid XML—DTDs and XML Schema

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Valid XML

- More often than not, applications that operate on XML data require the XML input data to conform to a **specific XML dialect**.
- 
  This requirement is more strict than just XML well-formedness.
- The (hard-coded) application logic relies on, *e.g.*,
  - the **presence** or absence of specifically named elements [attributes],
  - the **order** of child elements within an enclosing element,
  - attributes having exactly one of several **expected values**, ...
- If the input data fails to meet the requirements, results are often disastrous.

  Example: Transform element `amount` into attribute:

```
<bet gambler="doe"><amount>7</amount>...</bet>
                          ↓
<bet gambler="doe" amount="7">...</bet>
```

Stumbling Code

```
$ java foo input.xml
Calculate gambling results.....
Exception in thread "main" java.lang.NumberFormat
        at java.lang.Integer.parseInt(Integer.java:394)
        at java.lang.Integer.parseInt(Integer.java:476)
        at foo.getResult(foo.java:169)
        at foo.main(foo.java:214)

$ java bar input.xml
Exception in thread "main" java.lang.NullPointerException
        at bar.printGamblers(bar.java:186)
        at bar.main(bar.java:52)

$ java baz input.xml
Gambler John Doe lost 0.
Gambler Johnny Average lost 0.
Gambler Betty Bet lost 0.
Gambler Linda Loser lost 0.
Gambler Robert Johnson lost 0.

$
```

DTDs–Document Type Definitions

- The XML Recommendation[12] includes technology that enables applications to rigidly specify the XML **dialect** (the **document type**) they expect to see: **DTDs** (Document Type Definitions).
- XML parsers use the DTD to ensure that input data is not only well-formed but also conforms to the DTD (XML speak: input data is **valid**).

Valid XML documents $\subset$ Well-formed XML documents

- **Document validation** is critical, if
  - distinct organizations (B2B) need to share XML data
    $\Rightarrow$ also **share the DTDs**,
  - applications need to discover and explore yet unknown XML dialects,
  - high-speed XML throughput is required (once the input is validated, we can **abandon** a lot of **runtime checks**).

[12] `http://www.w3.org/TR/REC-xml`

- A document's DTD is directly attached to its XML text using a `DOCTYPE` declaration:

`DOCTYPE` Declaration

```
1  <?xml version="1.0"?>
2  <!DOCTYPE t d_e d_i>
3  <t>
4  ...
5  </t>
```

  - The `DOCTYPE` declaration follows the XML declaration (`<?xml...?>`) (comments `<!--...-->`, processing instructions `<?...?>` in between are OK).
  - The first parameter t of the `DOCTYPE` declaration is required to match the document's **root element** tag.
  - The document type definition itself consists of an **external subset** ($d_e \equiv$ `SYSTEM "`$\langle uri \rangle$`"`) as well as an **internal subset** ($d_i \equiv$ `[...]`), *i.e.*, embedded in the document itself).
  - Both subsets are optional. Should clashes occur, declarations in the internal subset override those in the external subset.

Example:

```
<!DOCTYPE strip
  SYSTEM "file://DilbertML.dtd" [ <!ENTITY phb "Pointy-Haired Boss"> ] >
```

external subset

internal subset

The `ELEMENT` Declaration

- The DTD `ELEMENT` declaration, in some sense, defines the *vocabulary* available in an XML dialect.
- Any XML element t to be used in the dialect needs to be introduced via

  <!ELEMENT t cm>

  - The **content model** cm of the element defines which **element content** is considered valid.
  - Whenever an application encounters a t element  **anywhere** in a valid document, it may assume that t's content conforms to cm.

Content model	Valid content
`ANY`	arbitrary well-formed XML content
`EMPTY`	no child elements allowed (attributes OK)
regular expression over tag names, `#PCDATA`, and constructors `,`, `|`, `+`, `*`, `?`	order and occurrence of child elements and text content must *match* the regular expression

N.B.

- A DTD with `<!ELEMENT` *t* `ANY >` gives the application no clue about *t*'s content. Use judiciously.
- A `<!ELEMENT` *t* `EMPTY >` forbids any content for *t* elements.
  Example: (X)HTML `img`, `br` tags:

XHTML 1.0 Strict DTD

```
1   <!ELEMENT img EMPTY>
2   ...
3   <!ELEMENT br EMPTY>
```

- **Regular expression** content models provide control over the exact order and occurence of child nodes below an element node:

Reg. exp.	**Semantics**
t (tag name)	child element with tag t
`#PCDATA`	text content (parsed character data)
c_1 , c_2	c_1 followed by c_2
c_1 | c_2	c_1 or, alternatively, c_2
c+	c, one or more times
c*	c, zero or more times
c?	optional c

Example (DilbertML):

DilbertML.dtd

```
1  <!ELEMENT panel (scene, bubbles*) >
2  <!ELEMENT scene (#PCDATA) >
3  <!ELEMENT bubbles (bubble+) >
4  <!ELEMENT bubble (#PCDATA) >
```

Example (modify `bubble` element so that we can use `<loud>...</loud>` and `<whisper>...</whisper>` to markup speech more accurately):

```
<bubble>E-mail <loud>two copies</loud> to me when you're done.</bubble>
```

DilbertML.dtd

```
1  <!ELEMENT panel   (scene, bubbles*) >
2  <!ELEMENT scene   (#PCDATA) >
3  <!ELEMENT bubbles (bubble+) >
4  <!ELEMENT bubble  (#PCDATA | loud | whisper)* >
5  <!ELEMENT loud    (#PCDATA) >
6  <!ELEMENT whisper (#PCDATA) >
```

- Element `bubble` is said to allow **mixed content** (text and element nodes), while `panel` and `bubbles` allow **element content** only. Elements `scene`, `loud`, `whisper` have **text content**.
- DTD restriction:
  The above example shows the only acceptable placements of `#PCDATA` in content models.

Element Content *vs.* Mixed Content

Element `bubbles` has *element content*: an XML parser will *not* report white space contained in a `bubbles` element to its underlying application.

Element `bubble` has *mixed content*: white space (`#PCDATA`) is regarded essential and thus reported to the appliation.

Dilbert.xml

```
1  <bubbles>↵
2  ␣␣<bubble>↵
3  ␣␣␣<loud>No coffee</loud>↵
4  ␣␣␣no research ...↵
5  ␣␣</bubble>↵
6  </bubbles>
```

SAX events

startElement (t ="bubbles")
startElement (t ="bubble")
characters (*buf* = "␣␣␣↵", *len* = 4)
startElement (t ="loud")
⋮
endElement (t ="bubble")
endElement (t ="bubbles")

Ex.: DTD and valid XML encoding academic titles

Academic.xml

```
1  <?xml version="1.0"?>
2  <!DOCTYPE academic [
3    <!ELEMENT academic (Prof?, (Dr, (rernat|emer|phil)*)?,
4                        Firstname, Middlename*, Lastname) >
5    <!ELEMENT Prof       EMPTY >
6    <!ELEMENT Dr         EMPTY >
7    <!ELEMENT rernat     EMPTY >
8    <!ELEMENT emer       EMPTY >
9    <!ELEMENT phil       EMPTY >
10   <!ELEMENT Firstname  (#PCDATA) >
11   <!ELEMENT Middlename (#PCDATA) >
12   <!ELEMENT Lastname   (#PCDATA) >
13 ]>
14
15 <academic>
16   <Prof/> <Dr/> <emer/>
17   <Firstname>Don</Firstname>
18   <Middlename>E</Middlename>
19   <Lastname>Knuth</Lastname>
20 </academic>
```

The ATTLIST Declaration

- Using the DTD ATTLIST declaration, validation of XML documents is extended to attributes.
- The ATTLIST declaration associates a list of attribute names a_i with their owning element named t:

ATTLIST Declaration

```
1  <!ATTLIST t
2     a1  τ1  d1
3       ...
4     an  τn  dn
5  >
```

  - The **attribute types** τ_i define which values are valid for attribute a_i.
  - The **defaults** d_i indicate if a_i is required or optional (and, if absent, if a default value should be assumed for a_i).
  - In XML, the attributes of an element are *unordered*. The ATTLIST declaration prescribes *no order* of attribute usage.

- Via **attribute types**, control over the valid attribute values can be exercised:

Attribute Type τ_i	**Semantics**
`CDATA`	**character data** (no <, but `<`, ...)
$(v_1 \mid v_2 \mid \ldots \mid v_m)$	**enumerated** literal values
`ID`	value is document-wide **unique identifier** for owner element
`IDREF`	**references** an element via its `ID` attribute

Example:

Academic.xml (fragment)

```
1  <!ELEMENT academic (Firstname, Middlename*, Lastname) >
2  <!ATTLIST academic
3    title (Prof|Dr) #REQUIRED
4    type CDATA #IMPLIED >
5  >
6
7  <academic title="Dr" type="rer.nat."> ... </academic>
```

- Attribute defaulting in DTDs:

Attribute Default d_i	**Semantics**
`#REQUIRED`	element **must have** attribute a_i
`#IMPLIED`	attribute a_i is **optional**
`"`v`"`	attribute a_i is **optional**, if absent, **default value** v for a_i is assumed
`#FIXED` v	attribute a_i is **optional**, if present, **must have** value v

Example:

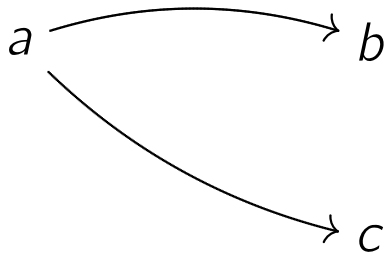
DilbertML.dtd (fragment)

```
1  <!DOCTYPE strip [
2   ...
3    <!ELEMENT characters (character+) >
4    <!ATTLIST characters
5      alphabetical (yes|no) "no" >   <!-- play safe -->
6    <!ELEMENT character (#PCDATA) >
7  ]>
```

Crossreferencing via ID and IDREF

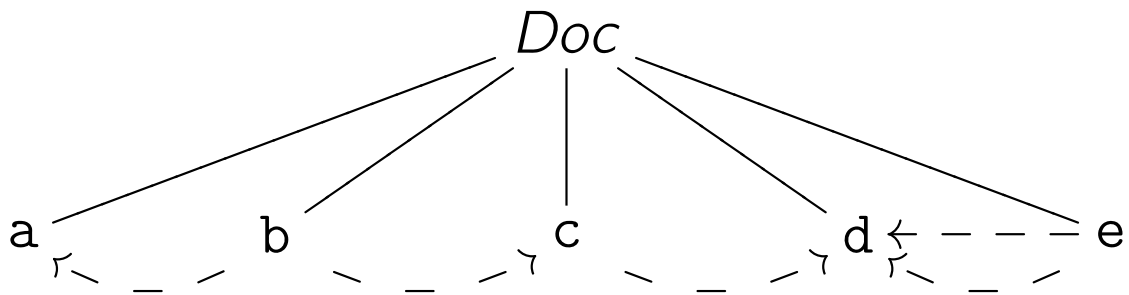
- Well-formed XML documents essentially describe tree-structured data.
- Attributes of type ID and IDREF may be used to encode **graph structures** in XML. A validating XML parser can check such a graph encoding for consistent connectivity.
- To establish a directed edge between two XML element nodes a and b

1. attach a unique **identifier** to node b (using an ID attribute), then
2. **refer** to b from a via this identifier (using an IDREF attribute),
3. for an outdegree > 1 (see below), use an IDREFS attribute.



Graph.xml

```
1  <?xml version="1.0"?>
2  <!DOCTYPE graph [
3    <!ELEMENT graph (node+) >
4    <!ELEMENT node ANY >        <!-- attach arbitrary data to a node -->
5    <!ATTLIST node
6      id    ID     #REQUIRED
7      edges IDREFS #IMPLIED >  <!-- we may have nodes with outdegree 0 -->
8  ]>
9
10 <graph>
11   <node id="A">a</node>
12   <node id="B" edges="A C">b</node>
13   <node id="C" edges="D">c</node>
14   <node id="D">d</node>
15   <node id="E" edges="D D">e</node>
16 </graph>
```



Example (Character references in DilbertML)

DilbertML.dtd (fragment)

```
1  <!DOCTYPE strip [
2    ...
3    <!ELEMENT character (#PCDATA) >
4    <!ATTLIST character
5      id       ID                      #REQUIRED >
6    <!ELEMENT bubble    (#PCDATA) >
7    <!ATTLIST bubble
8      speaker IDREF                    #REQUIRED
9      to      IDREFS                   #IMPLIED
10     tone    (angry|question|...) #IMPLIED >
11 ]>
```

Validation results (messages generated by Apache's Xerces):

- Setting attribute `to` to some random non-existent character identifier:
  `ID attribute 'yoda' was referenced but never declared`
- Using a non-enumerated value for attribute `tone`:
  `Attribute 'tone' does not match its defined enumeration list`

DilbertML.dtd

```
<!DOCTYPE strip [
  <!ELEMENT strip      (prolog, panels) >
  <!ATTLIST strip      copyright CDATA #IMPLIED
                       year      CDATA #IMPLIED >

  <!ELEMENT prolog     (series, author, characters) >

  <!ELEMENT series     (#PCDATA) >
  <!ATTLIST series     href CDATA #IMPLIED >

  <!ELEMENT author     (#PCDATA) >

  <!ELEMENT characters (character+) >
  <!ATTLIST characters alphabetical (yes|no) 'no' >

  <!ELEMENT character  (#PCDATA) >
  <!ATTLIST character  id ID #REQUIRED >

  <!ELEMENT panels     (panel+) >
  <!ATTLIST panels     length CDATA #IMPLIED >

  <!ELEMENT panel      (scene, bubbles*) >
  <!ATTLIST panel      no CDATA #IMPLIED >

  <!ELEMENT scene      (#PCDATA) >
  <!ATTLIST scene      visible IDREFS #IMPLIED >

  <!ELEMENT bubbles    (bubble+) >

  <!ELEMENT bubble     (#PCDATA) >
  <!ATTLIST bubble     speaker IDREF  #REQUIRED
                       to      IDREFS #IMPLIED
                       tone    (question|angry|screaming) #IMPLIED >
]>
```

Other DTD features

- **User-defined entities** via <!ENTITY *e d*> declarations (usage: &*e*;)

```
<!ENTITY phb "The Pointy-Haired Boss">
```

- **Parameter entities** ("DTD macros") via <!ENTITY % *e d*> (usage: %*e*;)

```
<!ENTITY ident "ID #REQUIRED">
...
<!ATTLIST character
  id %ident; >
```

- **Conditional sections** in DTDs via <![INCLUDE[...]]> and <![IGNORE[...]]>

```
<!ENTITY % withCharacterIDs "INCLUDE" >
<!ATTLIST bubble
  <![%withCharacterIDs;[
    speaker %ident;
    to      %ident;
  ]]>
  tone    (angry|question|...) #IMPLIED >
```

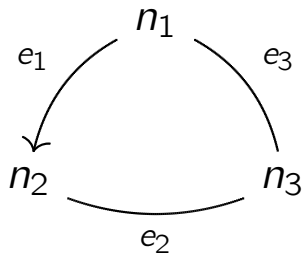
A "Real Life" DTD—GraphML

- GraphML[13] has been designed to provide a powerful and easy-to-use file format to represent arbitrary graphs.
  1. Graphs (element `graph`) are specified as lists of `nodes` and `edges`. Edges point from `source` to `target`.
  2. Nodes and edges may be annotated using arbitrary `descriptions` and `data`.
  3. Edges may be `directed` (and attribute `edgedefault` of `graph`).
  4. Edges may be attached to nodes at specific `ports` (north, west, ...).

Example:

GraphML.xml

```
1  <graphml>
2   <graph edgedefault="undirected">
3    <node id="n1"/>
4    <node id="n2"/>
5    <node id="n3"/>
6    <edge id="e1" source="n1" target="n2" directed="true"/>
7    <edge id="e2" source="n2" target="n3" directed="false"/>
8    <edge id="e3" source="n3" target="n1"/>
9   </graph>
10 </graphml>
```



[13] http://www.graphdrawing.org/

GraphML.dtd

```
1  <!-- ============================================================ -->
2  <!-- GRAPHML DTD (flat version) ================================= -->
3  <!-- file: graphml.dtd
4
5             SYSTEM "http://www.graphdrawing.org/dtds/graphml.dtd"
6
7             xmlns="http://www.graphdrawing.org/xmlns/graphml"
8             (consider these urls as examples)
9
10       ============================================================ -->
11
12
13 <!--============================================================-->
14 <!--elements of GRAPHML-->
15 <!--============================================================-->
16
17
18 <!ELEMENT graphml  ((desc)?,(key)*,((data)|(graph))*)>
19
20
21 <!ELEMENT locator EMPTY>
22 <!ATTLIST locator
23            xmlns:xlink   CDATA    #FIXED
```

```
24                          "http://www.w3.org/TR/2000/PR-xlink-20001220/"
25          xlink:href    CDATA    #REQUIRED
26          xlink:type    (simple) #FIXED    "simple"
27  >
28
29  <!ELEMENT desc (#PCDATA)>
30
31
32  <!ELEMENT graph    ((desc)?,((((data)|(node)|
33                               (edge)|(hyperedge))*)|(locator)))>
34  <!ATTLIST graph
35            id           ID                      #IMPLIED
36            edgedefault (directed|undirected) #REQUIRED
37  >
38
39  <!ELEMENT node   (desc?,(((data|port)*,graph?)|locator))>
40  <!ATTLIST node
41            id         ID      #REQUIRED
42  >
43
44  <!ELEMENT port ((desc)?,((data)|(port))*)>
45  <!ATTLIST port
46            name    NMTOKEN  #REQUIRED
```

```
47  >
48
49
50  <!ELEMENT edge ((desc)?,(data)*,(graph)?)>
51  <!ATTLIST edge
52            id         ID           #IMPLIED
53            source     IDREF        #REQUIRED
54            sourceport NMTOKEN      #IMPLIED
55            target     IDREF        #REQUIRED
56            targetport NMTOKEN      #IMPLIED
57            directed   (true|false) #IMPLIED
58  >
59
60
61  <!ELEMENT hyperedge  ((desc)?,((data)|(endpoint))*,(graph)?)>
62  <!ATTLIST hyperedge
63            id     ID      #IMPLIED
64  >
65
66  <!ELEMENT endpoint ((desc)?)>
67  <!ATTLIST endpoint
68            id    ID             #IMPLIED
69            node  IDREF          #REQUIRED
```

```
70          port  NMTOKEN         #IMPLIED
71          type  (in|out|undir) "undir"
72  >
73
74
75  <!ELEMENT key (#PCDATA)>
76  <!ATTLIST key
77          id  ID                                                  #REQUIRED
78          for (graph|node|edge|hyperedge|port|endpoint|all) "all"
79  >
80
81  <!ELEMENT data  (#PCDATA)>
82  <!ATTLIST data
83          key       IDREF         #REQUIRED
84          id        ID            #IMPLIED
85  >
86
87  <!--============================================================
88       end of graphml.dtd
89  ============================================================-->
```

Concluding remarks

- DTD syntax:

  Pro: compact, easy to understand
  Con: not in XML

- DTD functionality:
  - no distinguishable types (everything is character data)
  - no further value constraints (*e.g.*, cardinality of sequences)
  - no support for XML name spaces

- **From a database perspective, DTDs are a poor schema definition language.**

  (But: see XML Schema below. . .)

XML Schema

- With **XML Schema**[14], the W3C provides a **schema description language** for XML documents that goes way beyond the capabilities of the "native" DTD concept.
  Specifically:
  1. XML Schema **descriptions are valid XML documents** themselves.
  2. XML Schema provides a **rich set of built-in data types**.
     (Modelled after the SQL and Java type systems.)
  3. **Far-reaching control over the values** a data type can assume (**facets**).
  4. Users can extend this type system via **user-defined types**.
  5. XML element (and attribute) types may even be derived by **inheritance**.

✎ XML Schema *vs.* DTDs

Ad ①: Why would you consider this an advantage?
Ad ②: What are the data types supported by DTDs?

[14] `http://www.w3.org/TR/xmlschema-0/`

Some XML Schema Constructs

- **Declaring an element**

```
<xsd:element name="author"/>
```

No further typing specified: the `author` element may contain `string` values only.

- **Declaring an element with bounded occurence**

```
<xsd:element name="item" minOccurs="0" maxOccurs="unbounded"/>
```

Absence of `minOccurs`/`maxOccurs` implies *exactly once*.

- **Declaring a typed element**

```
<xsd:element name="year" type="xsd:date"/>
```

Content of `year` takes the format `YYYY-MM-DD`. Other **simple types**: `string`, `boolean`, `number`, `float`, `duration`, `time`, `base64Binary`, `AnyURI`, ...

- **Simple types** are considered **atomic** with respect to XML Schema (*e.g.*, the `YYYY` part of an `xsd:date` value has to be extracted by the XML application itself).

- Non-atomic **complex types** are built from simple types using **type constructors**.

Declaring sequenced content

```
1  <xsd:complexType name="Characters">
2    <xsd:sequence>
3      <xsd:element name="character" minOccurs="1" maxOccurs="unbounded"/>
4    </xsd:sequence>
5  </xsd:complexType>
6  <xsd:complexType name="Prolog">
7    <xsd:sequence>
8      <xsd:element name="series"/>
9      <xsd:element name="author"/>
10     <xsd:element name="characters" type="Characters"/>
11   </xsd:sequence>
12 </xsd:complexType>
13 <xsd:element name="prolog" type="Prolog"/>
```

An `xsd:complexType` may be used anonymously (no `name` attribute).

- With attribute `mixed="true"`, an `xsd:complexType` admits **mixed content**.

- New complex types may be **derived** from an existing (base) type.

Deriving a new complex type

```
1  <xsd:element name="newprolog">
2    <xsd:complexType>
3      <xsd:complexContent>
4        <xsd:extension base="Prolog">
5          <xsd:element name="colored" type="xsd:boolean"/>
6        </xsd:extension>
7      </xsd:complexContent>
8    </xsd:complexType>
9  </xsd:element>
```

- **Attributes** are declared within their owner element.

Declaring attributes

```
1  <xsd:element name="strip">
2    <xsd:attribute name="copyright"/>
3    <xsd:attribute name="year" type="xsd:gYear"/> ...
4  </xsd:element>
```

Other `xsd:attribute` modifiers: `use` (`required`, `optional`, `prohibited`), `fixed`, `default`.

- The validation of an XML document against an XML Schema declaration goes as far as peeking into the **lexical representation** of simple typed values.

Restricting the value space of a simple type (enumeration)

```
1  <xsd:simpleType name="Tone">
2    <xsd:restriction base="xsd:string">
3      <xsd:enumeration value="question"/>
4      <xsd:enumeration value="angry"/>
5      <xsd:enumeration value="screaming"/>
6    </xsd:restriction>
7  </xsd:simpleType>
```

Restricting the value space of a simple type (regular expression)

```
1  <xsd:simpleType name="AreaCode">
2    <xsd:restriction base="xsd:string">
3      <xsd:pattern value="0[0-9]+"/>
4      <xsd:minLength value="3"/>
5      <xsd:maxLength value="5"/>
6    </xsd:restriction>
7  </xsd:simpleType>
```

- Other **facets**: `length`, `maxInclusive`, `minExclusive`, ...

Other XML Schema Concepts

- **Fixed** and **default** element content,
- support for **null values**,
- uniqueness constraints, arbitrary **keys** (specified via XPath), local keys, key references, and referential integrity,
- ...

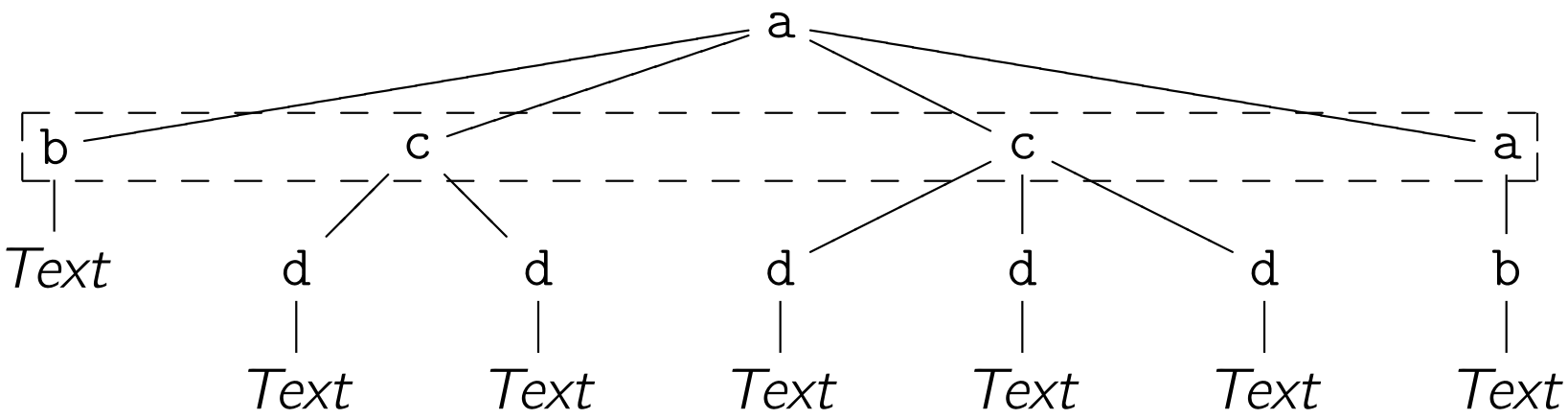
Validating XML Documents Against DTDs

- To validate against this DTD . . .

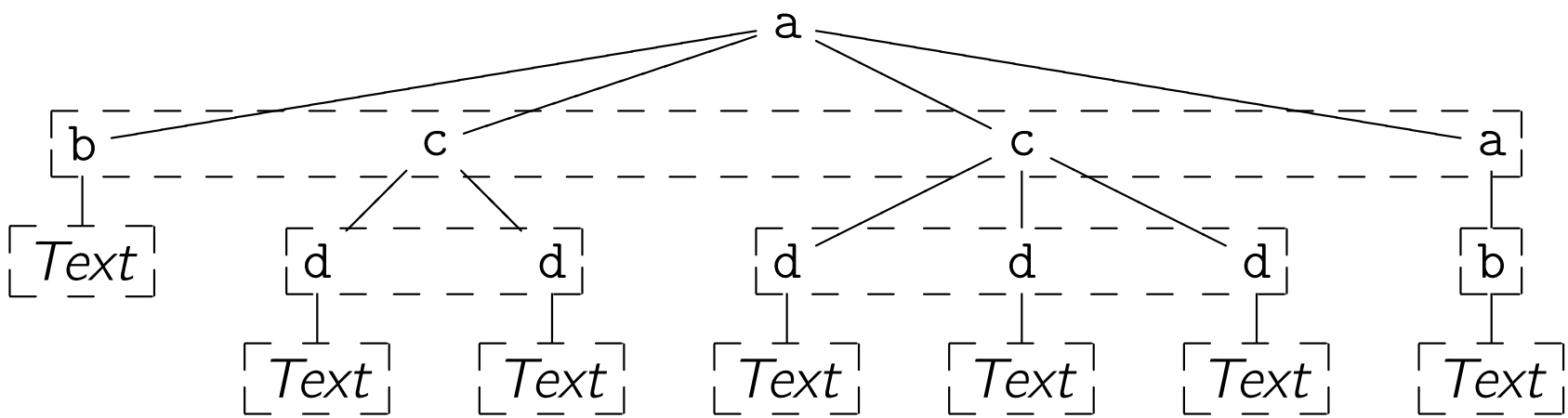
DTD featuring regular expression (RE) content models

```
1  <!DOCTYPE a [
2    <!ELEMENT a (b, c*, a?)>
3    <!ELEMENT b (#PCDATA)  >
4    <!ELEMENT c (d, d+)    >
5    <!ELEMENT d (#PCDATA)  >
6  ]>
```

. . . means to check that the **sequence of child nodes** for each element **matches** its RE content model:



- When, during RE matching, we encounter a child element t, we need to **recursively check t's content model** $cm(t)$ in the same fashion:



$$
\begin{aligned}
cm(\texttt{a}) &= \texttt{b,c}^*\texttt{,a?} \\
cm(\texttt{b}) &= \texttt{\#PCDATA} \\
cm(\texttt{c}) &= \texttt{d,d}^+ \\
cm(\texttt{d}) &= \texttt{\#PCDATA}
\end{aligned}
$$

✎ SAX and DTD validation?

1. Can we use SAX to drive this validation (= RE matching) process?
2. If so, which SAX events do we need to catch to implement this?

Regular Expressions

- To provide adequate support for SAX-based XML validation, we assume REs of the following structure:

RE	$=$	$\emptyset$	matches **nothing**
	$\mid$	ε	matches **empty** sequence of SAX events
	$\mid$	`#PCDATA`	matches *characters*$(\cdot)$
	$\mid$	t	matches *startElement*$(t, \cdot)$
	$\mid$	RE, RE	**concatenation**
	$\mid$	RE^+	**one-or-more repetitions**
	$\mid$	RE^*	**zero-or-more repetitions**
	$\mid$	$RE?$	**option**
	$\mid$	$RE \mid RE$	**alternative**
	$\mid$	(RE)	

-  $\emptyset$ and ε are *not* the same thing.
- In the *startElement*$(t, \cdot)$ callback we can process `<!ATTLIST` t `...>` declarations (not dicsussed here).

- Associated with each RE is the **regular language** $L(RE)$ (here: sets of sequences of SAX events) this RE **accepts**:

$$
\begin{aligned}
L(\emptyset) &= \emptyset \\
L(\varepsilon) &= \{\varepsilon\} \\
L(\texttt{\#PCDATA}) &= \{\mathit{characters}(\cdot)\} \\
L(t) &= \{\mathit{startElement}(t, \cdot)\}^{15} \\
L(RE_1, RE_2) &= \{s_1 s_2 \mid s_1 \in L(RE_1),\ s_2 \in L(RE_2)\} \\
L(RE^+) &= \bigcup_{i=1}^{\infty} L(RE^i) \\
L(RE^*) &= \bigcup_{i=0}^{\infty} L(RE^i) \\
L(RE?) &= \{\varepsilon\} \cup L(RE) \\
L(RE_1 \mid RE_2) &= L(RE_1) \cup L(RE_2)
\end{aligned}
$$

- **N.B.**: $RE^0 = \varepsilon$ and $RE^i = RE, RE^{i-1}$.

[15] To save trees, we will abbreviate this as $\{t\}$ from now on.

Example

- Which sequence of SAX events is matched by the RE $\texttt{\#PCDATA} \mid \texttt{b}^*$?

$$
\begin{aligned}
& L(\texttt{\#PCDATA} \mid \texttt{b}^*) \\
&= L(\texttt{\#PCDATA}) \cup L(\texttt{b}^*) \\
&= L(\texttt{\#PCDATA}) \cup \textstyle\bigcup_{i=0}^{\infty} L(\texttt{b}^i) \\
&= L(\texttt{\#PCDATA}) \cup L(\texttt{b}^0) \cup \textstyle\bigcup_{i=1}^{\infty} L(\texttt{b}^i) \\
&= L(\texttt{\#PCDATA}) \cup L(\texttt{b}^0) \cup L(\texttt{b}^1) \cup \textstyle\bigcup_{i=2}^{\infty} L(\texttt{b}^i) \\
&= L(\texttt{\#PCDATA}) \cup L(\texttt{b}^0) \cup L(\texttt{b}^1) \cup L(\texttt{b}^2) \cup \textstyle\bigcup_{i=3}^{\infty} L(\texttt{b}^i) \\
&= L(\texttt{\#PCDATA}) \cup L(\varepsilon) \cup L(\texttt{b}) \cup L(\texttt{b}, \texttt{b}^1) \cup \ldots \\
&= L(\texttt{\#PCDATA}) \cup L(\varepsilon) \cup L(\texttt{b}) \cup \{s_1 s_2 \mid s_1 \in L(\texttt{b}), s_2 \in L(\texttt{b}^1)\} \cup \ldots \\
&= \{\mathit{characters}(\cdot), \varepsilon, \texttt{b}, \texttt{bb}, \ldots\}
\end{aligned}
$$

✎ $L(\texttt{d},\texttt{d}^+) = \ ?$

Evaluating Regular Expressions (Matching)

- Now that we are this far, we know that matching a sequence of SAX events s against the content model of element t means to carry out the test

$$s \stackrel{?}{\in} L(cm(t)) \quad .$$

- $L(cm(t))$, however, might be infinite or otherwise too costly to construct inside our DTD validator.
- We thus follow a different path that avoids to enumerate $L(cm(t))$ at all.
- Instead, we will use the **derivative** $s\backslash RE$ of RE with respect to input event s:

$$L(s\backslash RE) \quad = \quad \{s' \mid s\,s' \in L(RE)\}$$

"$s\backslash RE$ matches everything matched by RE, with head s cut off."

- We can use the derivate operator $\backslash$ to develop a simple **RE matching procedure**.

  Suppose we are to match the SAX event sequence $s_1 s_2 s_3$ against RE:

$$\begin{aligned} s_1 s_2 s_3 \in L(RE) &\Leftrightarrow s_1 s_2 s_3 \varepsilon \in L(RE) \\ &\Leftrightarrow s_2 s_3 \varepsilon \in L(s_1 \backslash RE) \\ &\Leftrightarrow s_3 \varepsilon \in L\left(s_2 \backslash (s_1 \backslash RE)\right) \\ &\Leftrightarrow \varepsilon \in L\left(s_3 \backslash (s_2 \backslash (s_1 \backslash RE))\right) \quad . \end{aligned}$$

- We thus have solved our matching problem if
  1. we can efficiently **test for ε-containment** for a given RE, and
  2. we are able to **compute** $L(s \backslash RE)$ for any given input event s and any RE.

✎ Ad (1): Testing for ε's presence in a regular language.

Define a predicate (boolean function) $\mathit{nullable}(RE)$ such that

$$\mathit{nullable}(RE) \quad \Leftrightarrow \quad \varepsilon \in L(RE) \ .$$

$$\begin{aligned}
\mathit{nullable}(\emptyset) &= \mathit{false} \\
\mathit{nullable}(\varepsilon) &= \mathit{true} \\
\mathit{nullable}(\#\texttt{PCDATA}) &= \mathit{false} \\
\mathit{nullable}(t) &= \\
\mathit{nullable}(RE_1, RE_2) &= \\
\mathit{nullable}(RE^+) &= \\
\mathit{nullable}(RE^*) &= \\
\mathit{nullable}(RE?) &= \\
\mathit{nullable}(RE_1 \mid RE_2) &=
\end{aligned}$$

Example

Does $L(\#\texttt{PCDATA} \mid \texttt{b}^*)$ contain the empty SAX event sequence ε?

$$\begin{aligned} \mathit{nullable}(\#\texttt{PCDATA} \mid \texttt{b}^*) &= \mathit{nullable}(\#\texttt{PCDATA}) \vee \mathit{nullable}(\texttt{b}^*) \\ &= \mathit{false} \vee \mathit{true} \\ &= \mathit{true} \quad . \end{aligned}$$

✎ $\mathit{nullable}(\texttt{Prof?}, \texttt{Dr}, (\texttt{rernat} \mid \texttt{emer} \mid \texttt{phil})^+) = \ ?$

Ad ②: Note that the **derivative** $s\backslash$ is an operator on REs (to REs). We define it like follows and justify this definition on the next slides.

$$
\begin{aligned}
s\backslash\emptyset &= \emptyset \\
s\backslash\varepsilon &= \emptyset \\
s\backslash\#\texttt{PCDATA} &= \begin{cases} \varepsilon & \text{if } s = \mathit{characters}(\cdot) \\ \emptyset & \text{otherwise} \end{cases} \\
s\backslash t &= \begin{cases} \varepsilon & \text{if } s = \mathit{startElement}(t,\cdot) \quad //\bigstar \text{ recursively match } \mathit{cm}(t) \\ \emptyset & \text{otherwise} \end{cases} \\
s\backslash(RE_1, RE_2) &= \begin{cases} ((s\backslash RE_1), RE_2) \mid (s\backslash RE_2) & \text{if } \mathit{nullable}(RE_1) \\ (s\backslash RE_1), RE_2 & \text{otherwise} \end{cases} \\
s\backslash RE^+ &= (s\backslash RE), RE^* \\
s\backslash RE^* &= (s\backslash RE), RE^* \\
s\backslash RE? &= s\backslash RE \\
s\backslash(RE_1 \mid RE_2) &= (s\backslash RE_1) \mid (s\backslash RE_2)
\end{aligned}
$$

Correctness: Case Analysis I

To assess the correctness of this derivative construction $s\backslash RE = RE'$ we can systematically check all 9 cases for **language equivalence**, *i.e.*

$$L(s\backslash RE) \quad = \quad L(RE') \ .$$

1. $RE = \emptyset$:

$$\begin{aligned} L(s\backslash\emptyset) &= \{s' \mid s\,s' \in L(\emptyset)\} \\ &= \{s' \mid s\,s' \in \emptyset\} \\ &= \emptyset \\ &= L(\emptyset). \end{aligned}$$

Correctness: Case Analysis II

2. $RE = \varepsilon$:

$$\begin{aligned} L(s\backslash\varepsilon) &= \{s' \mid s\,s' \in L(\varepsilon)\} \\ &= \{s' \mid s\,s' \in \{\varepsilon\}\} \\ &= \emptyset \\ &= L(\emptyset). \end{aligned}$$

3. $RE = \#\texttt{PCDATA}, s = \mathit{characters}(\cdot)$:

$$\begin{aligned} L(\mathit{characters}(\cdot)\backslash\#\texttt{PCDATA}) &= \{s' \mid \mathit{characters}(\cdot)\,s' \in L(\#\texttt{PCDATA})\} \\ &= \{s' \mid \mathit{characters}(\cdot)\,s' \in \{\mathit{characters}(\cdot)\}\} \\ &= \{\varepsilon\} \\ &= L(\varepsilon). \end{aligned}$$

Correctness: Case Analysis III

$RE = \#\texttt{PCDATA}, s \neq \mathit{characters}(\cdot)$:

$$\begin{aligned} L(s\backslash\#\texttt{PCDATA}) &= \{s' \mid s\,s' \in L(\#\texttt{PCDATA})\} \\ &= \{s' \mid s\,s' \in \{\mathit{characters}(\cdot)\}\} \\ &= \emptyset \\ &= L(\emptyset). \end{aligned}$$

4. $RE = t$. Analogous to ③.
5. $RE = RE_1, RE_2$, $\mathit{nullable}(RE_1) = \mathit{false}$:

$$\begin{aligned} L(s\backslash(RE_1, RE_2)) &= \{s' \mid s\,s' \in L(RE_1, RE_2)\} \\ &= \{s' \mid s' \in L((s\backslash RE_1), RE_2)\} \\ &= L((s\backslash RE_1), RE_2). \end{aligned}$$

Correctness: Case Analysis IV

$RE = RE_1, RE_2$, $\mathit{nullable}(RE_1) = \mathit{true}$:

$$
\begin{aligned}
L(s\backslash(RE_1, RE_2)) &= \{s' \mid s\,s' \in L(RE_1, RE_2)\} \\
&= \{s' \mid s\,s' \in L(RE_2) \vee s\,s' \in L(RE_1, RE_2)\} \\
&= \{s' \mid s' \in L(s\backslash RE_2) \vee s' \in L((s\backslash RE_1), RE_2)\} \\
&= \{s' \mid s' \in L(s\backslash RE_2)\} \cup \{s' \mid s' \in L((s\backslash RE_1), RE_2)\} \\
&= L(s\backslash RE_2) \cup L((s\backslash RE_1), RE_2) \\
&= L\left((s\backslash RE_2) \mid ((s\backslash RE_1), RE_2)\right).
\end{aligned}
$$

Correctness: Case Analysis V

6. $RE = RE_1 \mid RE_2$:

$$
\begin{aligned}
L(s\backslash(RE_1 \mid RE_2)) &= \{s' \mid s\,s' \in L(RE_1 \mid RE_2)\} \\
&= \{s' \mid s\,s' \in L(RE_1) \cup L(RE_2)\} \\
&= \{s' \mid s\,s' \in L(RE_1)\} \cup \{s' \mid s\,s' \in L(RE_2)\} \\
&= \{s' \mid s' \in L(s\backslash RE_1)\} \cup \{s' \mid s' \in L(s\backslash RE_2)\} \\
&= L(s\backslash RE_1) \cup L(s\backslash RE_2) \\
&= L\left((s\backslash RE_1) \mid (s\backslash RE_2)\right).
\end{aligned}
$$

Correctness: Case Analysis VI

7. $RE = RE^*$, $\mathit{nullable}(RE) = \mathit{false}$:

$$\begin{aligned} L(s\backslash RE^*) &= L(s\backslash(\varepsilon \mid (RE, RE^*))) \\ &= L(s\backslash\varepsilon) \cup L(s\backslash(RE, RE^*)) \\ &= L(s\backslash(RE, RE^*)) \\ &= L((s\backslash RE), RE^*). \end{aligned}$$

$RE = RE^*$, $\mathit{nullable}(RE) = \mathit{true}$:

$$\begin{aligned} L(s\backslash RE^*) &= L(s\backslash(\varepsilon \mid (RE, RE^*))) \\ &= L((s\backslash\varepsilon) \mid (s\backslash(RE, RE^*))) \\ &= L(\emptyset \mid (s\backslash(RE, RE^*))) \\ &= L(s\backslash(RE, RE^*)) \\ &= L((s\backslash RE^*) \mid ((s\backslash RE), RE^*)) \\ &= L(s\backslash RE^*) \cup L((s\backslash RE), RE^*) \\ &= L((s\backslash RE), RE^*). \end{aligned}$$

Correctness: Case Analysis VII

8. $RE = RE^+$. Follows from $RE^+ = RE, RE^*$.
9. $RE = RE?$. Follows from $RE? = \varepsilon \mid RE$.

✎ Matching SAX events against an RE

RE content model b,c*,a? is to be matched against SAX events bcca.[16]

To validate,

1. construct the corresponding derivative $RE' = \mathtt{a}\backslash\ (\mathtt{c} \setminus (\mathtt{c} \setminus (\mathtt{b} \setminus (\mathtt{b},\mathtt{c}^*,\mathtt{a}?))))$,
2. then test *nullable*(RE').

Hint: To simplify phase ①, use the following **laws**, valid for REs in general:

$$\begin{array}{lcl} \varepsilon^* & = & \varepsilon \\ \emptyset^* & = & \varepsilon \\ \varepsilon^+ & = & \varepsilon \\ \emptyset^+ & = & \emptyset \\ \varepsilon? & = & \varepsilon \\ \emptyset? & = & \varepsilon \end{array} \qquad \begin{array}{lcl} \varepsilon, RE & = & RE \\ \emptyset, RE & = & \emptyset \\ RE, \varepsilon & = & RE \\ RE, \emptyset & = & \emptyset \\ \emptyset \mid RE & = & RE \\ RE \mid \emptyset & = & RE \end{array}$$

[16] Actual event sequence:
startElement(b,·), *startElement*(c,·), *startElement*(c,·), *startElement*(a,·).

Plugging It All Together

The following SAX callbacks use the aforementioned RE matching techniques to (partially) implement DTD validation **while parsing** the input XML document:

The input DTD (declaring the content models $cm(\cdot)$) is

```
<!DOCTYPE r [ ... ]>
```

startDocument()

$S.empty()$;
$RE \leftarrow cm(r)$;
return;

startElement$(t, \cdot)$

$RE \leftarrow t \backslash RE$;
$S.push(RE)$;
$RE \leftarrow cm(t)$;
return;

endElement(t)

if *nullable*(RE) then
  $RE \leftarrow S.pop()$;
else
  ★ FAIL ★;
return;

characters$(\cdot)$

$RE \leftarrow$ #PCDATA$\backslash RE$;
return;

endDocument()

★ OK ★;

N.B. Stack S is used to suspend [resume] the RE matching for a specific element node whenever SAX descends [ascends] the XML document tree.