



Chapter 8

Evaluation of Relational Operators

Implementing the Relational Algebra

Architecture and Implementation of Database Systems

Summer 2016

Relational Query Engines

Operator Selection

Selection (σ)

Selectivity

Conjunctive Predicates

Disjunctive Predicates

Projection (π)

Join (\bowtie)

Nested Loops Join

Block Nested Loops Join

Index Nested Loops Join

Sort-Merge Join

Hash Join

Operator Pipelining

Volcano Iterator Model





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Relational Query Engines

- In many ways, a DBMS's **query engine** compares to virtual machines (e.g., the Java VM):

Relational Query Engine	Virtual Machine (VM)
Operators of the relational algebra	Primitive VM instructions
Operates over streams of rows	Acts on object representations
Operator network (tree/DAG)	Sequential program (with branches, loops)
Several equivalent variants of an operator	Compact instruction set



Equivalent operator variants

Instead of a single \bowtie operator, a typical DBMS query engine features equivalent variants \bowtie' , \bowtie'' ,

What would **equivalent** mean in the context of the relational model?

Operator Variants

- Specific operator variants may be tailored to exploit **physical properties** of its input or the current system state:
 - ① The **presence or absence of indexes** on the input file(s),
 - ② the **sortedness** of the input file(s),
 - ③ the **size** of the input file(s),
 - ④ the **available space in the buffer pool**,
 - ⑤ the **buffer replacement policy**,
 - ⑥ ...

Physical operators

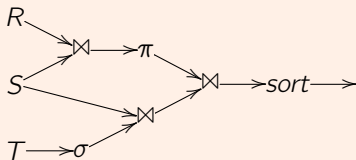
The variants (\bowtie' , \bowtie'') are thus referred to **physical operators**. They implement the **logical operators** of the relational algebra.

- The **query optimizer** is in charge to perform optimal (or, reasonable) **operator selection** (much like the instruction selection phase in a programming language compiler).

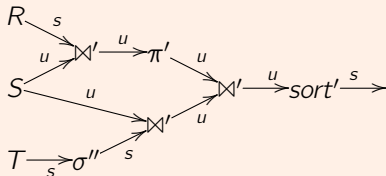


Operator Selection

Initial, logical operator network ("plan")



Physical plan with (un)sortedness annotations (u/s)



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- Selectivity
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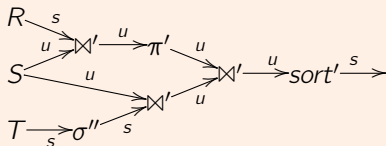
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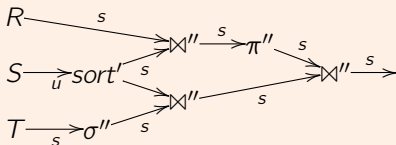
Plan Rewriting

Physical plan with (un)sortedness annotations (u/s)



- **Rewrite** the plan to exploit that the \oplus'' variant of operator \oplus can benefit from/preserve sortedness of its input(s):

Rewritten physical plan (preserve equivalence!)



Selection (σ)—No Index, Unsorted Data

- **Selection** (σ_p) reads an input file R_{in} of records and writes those records satisfying predicate p into the output file:

Selection

```
1 Function:  $\sigma(p, R_{in}, R_{out})$   
2  $out \leftarrow \text{createFile}(R_{out}) ;$   
3  $in \leftarrow \text{openScan}(R_{in}) ;$   
4 while ( $r \leftarrow \text{nextRecord}(in)$ )  $\neq \langle \text{EOF} \rangle$  do  
5   | if  $p(r)$  then  
6   |   |  $\text{appendRecord}(out, r) ;$   
7  $\text{closeFile}(out) ;$ 
```



Selection (σ)—No Index, Unsorted Data

Remarks:

- Reading the special “record” $\langle \text{EOF} \rangle$ from a file via `nextRecord()` indicates that all its record have been retrieved (scanned) already.
- This simple procedure does **not require** r_{in} **to come with any special physical properties** (the procedure is exclusively defined in terms of heap files).
- In particular, **predicate** p may be **arbitrary**.



Selection (σ)—No Index, Unsorted Data

- We can summarize the characteristics of this implementation of the selection operator as follows:

Selection (σ)—no index, unsorted data

$\sigma_p (R)$

input access ¹	file scan (openScan) of R
prerequisites	none (p arbitrary, R may be a heap file)
I/O cost	$\underbrace{N_R}_{\text{input cost}} + \underbrace{sel(p) \cdot N_R}_{\text{output cost}}$

- N_R denotes the **number of pages** in file R , $|R|$ denotes the **number of records**
(if p_R records fit on one page, we have $N_R = \lceil |R|/p_R \rceil$)

¹Also known as **access path** in the literature and text books.



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Aside: Selectivity

- $sel(p)$, the **selectivity of predicate** p , is the fraction of records satisfying predicate p :

$$0 \leq sel(p) = \frac{|\sigma_p(R)|}{|R|} \leq 1$$

 Selectivity examples

What can you say about the following selectivities?

- 1 $sel(true)$
- 2 $sel(false)$
- 3 $sel(A = 0)$

DB2 Estimated selectivities

IBM DB2 reports (estimated) selectivities in the operators details of, e.g., its IXSCAN operator.

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Selection (σ)—Matching Predicates with an Index

- A selection on input file R can be sped up considerably if an index has been defined and that **index matches predicate** p .
- The matching process depends on p itself as well as on the index type. If there is no immediate match but p is **compound**, a sub-expression of p may still find a **partial match**. Residual predicate evaluation work may then remain.

 When does a predicate match a sort key?

Assume R is tree-indexed on attribute A in ascending order.
Which of the selections below can benefit from the index on R ?

- 1 $\sigma_{A=42} (R)$
- 2 $\sigma_{A<42} (R)$
- 3 $\sigma_{A>42 \text{ AND } A<100} (R)$
- 4 $\sigma_{A>42 \text{ OR } A>100} (R)$
- 5 $\sigma_{A>42 \text{ AND } A<32} (R)$
- 6 $\sigma_{A>42 \text{ AND } B=10} (R)$
- 7 $\sigma_{A>42 \text{ OR } B=10} (R)$

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Selection (σ)—B⁺-tree Index

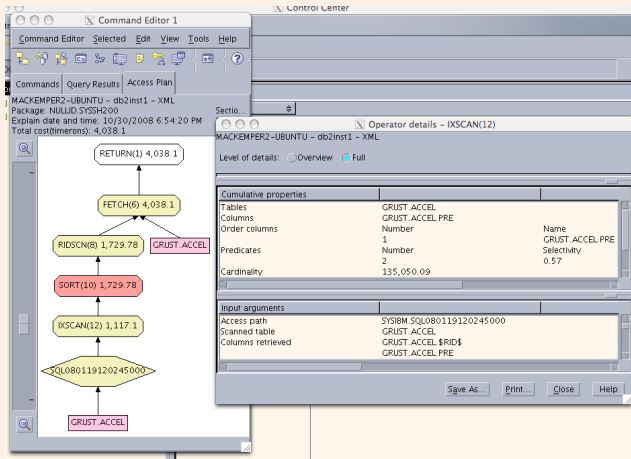
- A **B⁺-tree index** on R whose key **matches** the selection predicate p is clearly the superior method to evaluate $\sigma_p (R)$:
 - **Descend the B⁺-tree** to retrieve the first index entry to satisfy p . If the index is **clustered**, access that record on its page in R and continue to scan inside R .
 - If the index is **unclustered** and $sel(p)$ indicates a large number of qualifying records, it pays off to
 - ① read the matching index entries $k* = \langle k, rid \rangle$ in the sequence set,
 - ② sort those entries on their rid field,
 - ③ and then access the pages of R in sorted rid order.
 Note that lack of clustering is a minor issue if $sel(p)$ is close to 0.

DB2. Accessing unclustered B⁺-trees

IBM DB2 uses physical operator quadruple
IXSCAN/SORT/RIDSCN/FETCH to implement the above strategy.

Selection (σ)—B⁺-tree Index

The IXSCAN/SORT/RIDSCN/FETCH quadruple



- Note: Selectivity of predicate estimated as 57% (table accel has 235,501 rows).



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Selection (σ)—B⁺-tree Index



Selection (σ)—clustered B⁺-tree index

$\sigma_p(R)$

input access access of B⁺-tree on R , then sequence set scan

prerequisites clustered B⁺-tree on R with key k , p matches key k

I/O cost

$$\underbrace{\approx 3}_{\text{B}^+\text{-tree acc.}} + \underbrace{sel(p) \cdot N_R}_{\text{sorted scan}} + \underbrace{sel(p) \cdot N_R}_{\text{output cost}}$$

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Selection (σ)—Hash Index, Equality Predicate

- A selection predicate p **matches an hash index** only if p contains a term of the form $A = c$ (c constant, assuming the hash index has been built over column A).
- We are directly led to the bucket(s) of qualifying records and pay I/O cost only for this direct access². Note that $sel(p)$ is likely to be close to 0 for many equality predicates.

Selection (σ)—hash index, equality predicate $\sigma_p(R)$

input access	hash table on R
prerequisites	r_{in} hashed on key A , p has term $A = c$
I/O cost	$\underbrace{sel(p) \cdot N_R}_{\text{bucket access}} + \underbrace{sel(p) \cdot N_R}_{\text{output cost}}$

²Remember that this may include access cost for the pages of an overflow chain hanging off the primary bucket page.

Selection (σ)—Conjunctive Predicates

- Indeed, selection operations with simple predicates like $\sigma_{A\theta C}(R)$ are a special case only.
- We somehow need to deal with **complex predicates**, built from **simple comparisons** and the **Boolean connectives** AND and OR.
- Matching a selection predicate with an index can be extended to cover the case where predicate p has a **conjunctive form**:

$$\underbrace{A_1 \theta_1 C_1}_{\text{conjunct}} \text{ AND } A_2 \theta_2 C_2 \text{ AND } \dots \text{ AND } A_n \theta_n C_n .$$

- Here, each **conjunct** is a simple comparison ($\theta_i \in \{=, <, >, <=, >=\}$).
- An index with a multi-attribute key may match the *entire* complex predicate.



Selection (σ)—Conjunctive Predicates

Matching a multi-attribute hash index

Consider a hash index for the multi-attribute key $k = (A, B, C)$, *i.e.*, all three attributes are input to the hash function.

Which conjunctive predicates p would **match** this type of index?

Conjunctive predicate match rule for hash indexes

A **conjunctive predicate** p **matches a (multi-attribute) hash index** with key $k = (A_1, A_2, \dots, A_n)$, if p **covers the key**, *i.e.*,

$$p \equiv A_1 = c_1 \text{ AND } A_2 = c_2 \text{ AND } \dots \text{ AND } A_n = c_n \text{ AND } \phi .$$

The residual conjunct ϕ is not supported by the index itself and has to be **evaluated after index retrieval**.



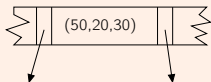
Selection (σ)—Conjunctive Predicates

Matching a multi-attribute B⁺-tree index

Consider a B⁺-tree index for the multi-attribute key $k = (A, B, C)$, *i.e.*, the B⁺-tree nodes are searched/inserted in lexicographic order w.r.t. these three attributes:

$$\begin{aligned}k_1 < k_2 &\equiv A_1 < A_2 \vee \\ &\quad (A_1 = A_2 \wedge B_1 < B_2) \vee \\ &\quad (A_1 = A_2 \wedge B_1 = B_2 \wedge C_1 < C_2)\end{aligned}$$

Excerpt of an inner B⁺-tree node (separator):



Which conjunctive predicates p would **match** this type of index?



Selection (σ)—Conjunctive Predicates



Conjunctive predicate match rule for B^+ -tree indexes

A **conjunctive predicate** ρ matches a **(multi-attribute) B^+ -tree index** with key $k = (A_1, A_2, \dots, A_n)$, if ρ is a **prefix of the key**, *i.e.*,

$$\rho \equiv A_1 \theta_1 c_1 \text{ AND } \phi$$

$$\rho \equiv A_1 \theta_1 c_1 \text{ AND } A_2 \theta_2 c_2 \text{ AND } \phi$$

$$\vdots$$

$$\rho \equiv A_1 \theta_1 c_1 \text{ AND } A_2 \theta_2 c_2 \text{ AND } \dots \text{ AND } A_n \theta_n c_n \text{ AND } \phi$$

- **Note:** Whenever a multi-attribute hash index matches a predicate, so does a B^+ -tree over the same key.

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- If the system finds that a conjunctive predicate does not match a single index, its (smaller) **conjuncts may nevertheless match distinct indexes**.

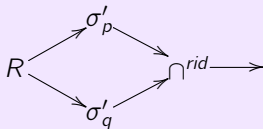
Example (Partial predicate match)

The conjunctive predicate in $\sigma_{p \text{ AND } q}(R)$ does not match an index, but both conjuncts p , q do.

A typical optimizer might thus decide to transform the original query

$$R \longrightarrow \sigma_{p \text{ AND } q} \longrightarrow$$

into



Here, \cap^{rid} denotes a **set intersection operator defined by rid equality** (IBM DB2: IXAND).

Selection (σ)—Conjunctive Predicates



Selectivity of conjunctive predicates

What can you say about the selectivity of the conjunctive predicate p AND q ?

$$sel(p \text{ AND } q) =$$

Now assume $p \equiv \text{AGE} \leq 16$ and $q \equiv \text{SALARY} > 5000$.
Reconsider your proposal above.

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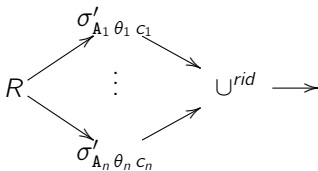
Selection (σ)—Disjunctive Predicates

- Choosing a reasonable execution plan for **disjunctive selection** of the general form

$$A_1 \theta_1 c_1 \quad \text{OR} \quad A_2 \theta_2 c_2 \quad \text{OR} \quad \dots \quad \text{OR} \quad A_n \theta_n c_n$$

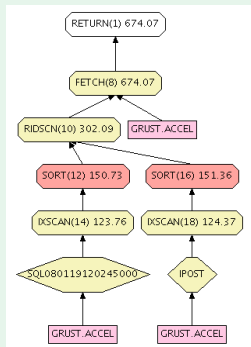
is much harder:

- We are forced to **fall back to a naive file scan based evaluation** as soon only a **single term does not match** an index.
- If **all terms are matched** by indexes, we can exploit a **rid-based set union \cup^{rid}** to improve the plan:



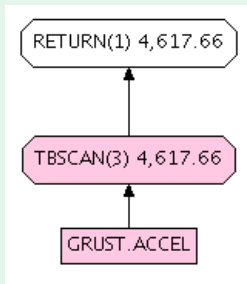
Selection (σ)—Disjunctive Predicates

DB2. Selective disjunctive predicate



Note: Multi-input RIDSCN operator.

DB2. Non-selective disjunctive predicate



Note: Presence of indexes ignored.



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Selectivity of disjunctive predicates

What can you say about the selectivity of the disjunctive predicate p OR q ?

$$sel(p \text{ OR } q) =$$

Projection (π)

- **Projection** (π_{ℓ}) modifies each record in its input file and cuts off any field not listed in the attribute list ℓ :

Relational projection

$$\pi_{A,B} \left(\begin{array}{|c|c|c|} \hline A & B & C \\ \hline 1 & 'foo' & 3 \\ \hline 1 & 'bar' & 2 \\ \hline 1 & 'foo' & 2 \\ \hline 1 & 'bar' & 0 \\ \hline 1 & 'foo' & 0 \\ \hline \end{array} \right) \stackrel{\textcircled{1}}{=} \begin{array}{|c|c|} \hline A & B \\ \hline 1 & 'foo' \\ \hline 1 & 'bar' \\ \hline 1 & 'foo' \\ \hline 1 & 'bar' \\ \hline 1 & 'foo' \\ \hline \end{array} \stackrel{\textcircled{2}}{=} \begin{array}{|c|c|} \hline A & B \\ \hline 1 & 'foo' \\ \hline 1 & 'bar' \\ \hline \end{array}$$

- In general, the size of the resulting file will only be a fraction of the original input file:
 - ① any unwanted fields (here: C) have been thrown away, and
 - ② optionally **duplicates removed** (SQL: DISTINCT).



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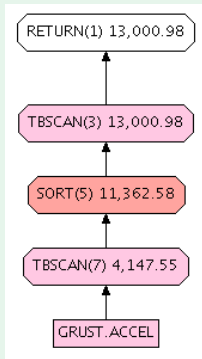
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Projection (π)—Duplicate Elimination, Sorting

- **Sorting** is one obvious preparatory step to facilitate duplicate elimination: records with all fields equal will end up adjacent to each other.

DB2. Implementing DISTINCT



- One benefit of sort-based projection is that operator π_{ℓ} will write a sorted output file, *i.e.*,

$$R \xrightarrow{?} \pi_{\ell}^{\text{sort}} \xrightarrow{s}$$

Sort ordering?

What would be the correct ordering θ to apply in the case of duplicate elimination?

Projection (π)—Duplicate Elimination, Hashing

- If the DBMS has a fairly large number of buffer pages (B , say) to spare for the $\pi_\ell(R)$ operation, a **hash-based** projection may be an efficient alternative to sorting:

Hash-based projection π_ℓ : partitioning phase

- 1 Allocate all B buffer pages. One page will be the **input buffer**, the remaining $B - 1$ pages will be used as **hash buckets**.
- 2 Read the file R page-by-page, for each record r : cut off fields not listed in ℓ .
- 3 For each such record, apply hash function $h_1(r) = h(r) \bmod (B - 1)$ —which depends on **all remaining fields of r** —and store r in hash bucket $h_1(r)$. (Write the bucket to disk if full.)



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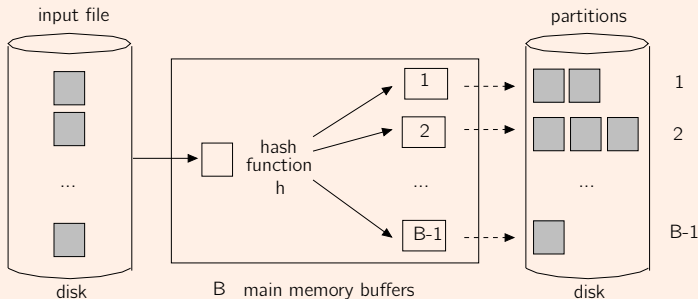
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Projection (π)—Hashing

Hash-based projection π_ℓ : partitioning phase



- After partitioning, duplicate elimination becomes an **intra-partition** problem only: two identical records have been mapped to the same partition:

$$h_1(r) = h_1(r') \quad \Leftarrow \quad r = r' .$$



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Projection (π)—Hashing

Hash-based projection π_ℓ : duplicate elimination phase

- 1 For each partition, read each partition page-by-page (possibly in parallel).
- 2 To each record, apply hash function $h_2 \neq h_1$ to all record fields.
- 3 Only if two records **collide** w.r.t. h_2 , check if $r = r'$. If so, discard r' .
- 4 After the entire partition has been read in, append all hash buckets to the result file (which will be free of duplicates).

Huge partitions?

Note: Works efficiently only if duplicate elimination phase can be **performed in the buffer** (main memory).

What to do if partition size exceeds buffer size?



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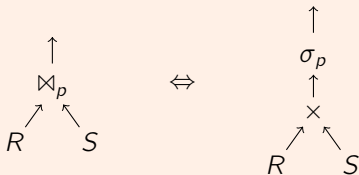
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The Join Operator (\bowtie)

The **join operator** \bowtie_p is actually a short-hand for a combination of **cross product** \times and **selection** σ_p .

Join vs. Cartesian product



One way to implement \bowtie_p is to follow this equivalence:

- 1 Enumerate and concatenate all records in the cross product of r_1 and r_2 .
- 2 Then pick those that satisfy p .

More advanced algorithms try to avoid the obvious inefficiency in Step 1 (the size of the intermediate result is $|R| \cdot |S|$).

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Nested Loops Join

The **nested loops join** is the straightforward implementation of the σ - \times combination:

Nested loops join

```

1 Function: nljoin( $R, S, p$ )
   /* outer relation  $R$  */ */
2 foreach record  $r \in R$  do
   /* inner relation  $S$  */ */
3   foreach record  $s \in S$  do
4     /*  $\langle r, s \rangle$  denotes record concatenation */ */
5     if  $\langle r, s \rangle$  satisfies  $p$  then
6       append  $\langle r, s \rangle$  to result

```

Let N_R and N_S the number of **pages** in R and S ; let p_R and p_S be the number of records per page in R and S .

The **total number of disk reads** then is

$$N_R + \underbrace{p_R \cdot N_R}_{\text{\# tuples in } R} \cdot N_S .$$

Nested Loops Join: I/O Behavior

The **good news** about `nljoin()` is that it needs only **three pages** of buffer space (two to read R and S , one to write the result).

The **bad news** is its enormous I/O cost:

- Assuming $p_R = p_S = 100$, $N_R = 1000$, $N_S = 500$, we need to read $1000 + (5 \cdot 10^7)$ disk pages.
- With an access time of 10 ms for each page, this join would take 140 hours!
- Switching the role of R and S to make S (the smaller one) the **outer relation** does not bring any significant advantage.

Note that reading data page-by-page (even tuple-by-tuple) means that **every** I/O suffers the disk latency penalty, even though we process both relations in sequential order.



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Block Nested Loops Join

- Again we can **save random access cost** by reading R and S in **blocks** of, say, b_R and b_S pages.

Block nested loops join

```
1 Function: block_nljoin( $R, S, p$ )  
2 foreach  $b_R$ -sized block in  $R$  do  
3   foreach  $b_S$ -sized block in  $S$  do  
4     /* performed in the buffer                               */  
     find matches in current  $R$ - and  $S$ -blocks and append  
     them to the result ;
```

- R is still read once, but now with only $\lceil N_R/b_R \rceil$ disk seeks.
- S is scanned only $\lceil N_R/b_R \rceil$ times now, and we need to perform $\lceil N_R/b_R \rceil \cdot \lceil N_S/b_S \rceil$ disk seeks to do this.



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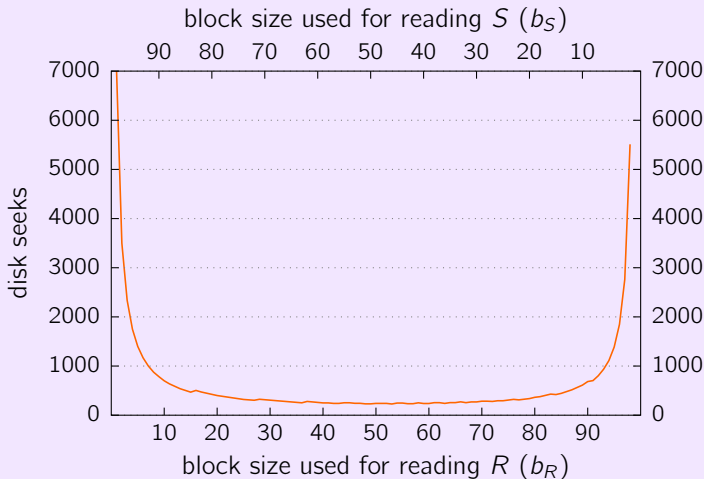
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Choosing b_R and b_S

Example: Buffer pool with $B = 100$ frames, $N_R = 1000$,
 $N_S = 500$:

Example (Choosing b_r and b_s)



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In-Memory Join Performance

- Line 4 in `block_nljjoin` (R, S, p) implies an **in-memory join** between the R - and S -blocks currently in memory.
- Building a hash table over the R -block can speed up this join considerably.

Block nested loops join: build hash table from outer row block

```
1 Function: block_nljjoin' ( $R, S, p$ )
2 foreach  $b_R$ -sized block in  $R$  do
3   build an in-memory hash table  $H$  for the current  $R$ -block ;
4   foreach  $b_S$ -sized block in  $S$  do
5     foreach record  $s$  in current  $S$ -block do
6       probe  $H$  and append matching  $\langle r, s \rangle$  tuples to result ;
```

- Note that this optimization only helps **equi-joins**.



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Index Nested Loops Join

The **index nested loops join** takes advantage of an index on the **inner** relation (swap *outer* \leftrightarrow *inner* if necessary):

Index nested loops join

```

1 Function: index_nljoin ( $R, S, p$ )
2 foreach record  $r \in R$  do
3   scan  $S$ -index using (key value in)  $r$  and concatenate  $r$  with all
4   matching tuples  $s$  ;
   append  $\langle r, s \rangle$  to result ;

```

- The index must **match** the join condition p .
 - Hash indices, *e.g.*, only support equality predicates.
 - Remember the discussion about composite keys in B^+ -trees.
- Such predicates are also called **sargable** (*sarg*: search argument \nearrow Selinger *et al.*, *SIGMOD 1979*)

Index Nested Loop Join: I/O Behavior

For each record in R , we use the index to find matching S -tuples. While searching for matching S -tuples, we incur the following I/O costs **for each tuple** in R :

- 1 **Access** the index to find its first matching entry: N_{idx} I/Os.
- 2 **Scan** the index to retrieve **all** n matching *rids*. The I/O cost for this is typically negligible (locality in the index).
- 3 **Fetch** the n matching S -tuples from their data pages.
 - For an **unclustered** index, this requires n I/Os.
 - For a **clustered** index, this only requires $\lceil n/p_S \rceil$ I/Os.

Note that (due to 2 and 3), the cost of an index nested loops join becomes **dependent on the size of the join result**.



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If the index is a **B⁺-tree index**:

- A **single** index access requires the inspection of h pages.³
- If we **repeatedly** probe the index, however, most of these are **cached** by the buffer manager.
- The effective value for N_{idx} is around 1–3 I/Os.

If the index is a **hash index**:

- Caching will not help here (no locality in accesses to hash table).
- A typical value for N_{idx} is 1.2 I/Os (> 1 due to overflow pages).

Overall, the use of an index (over, e.g., a block nested loops join) pays off if the join is **selective** (picks out only few tuples from a big table).

³ h : B⁺-tree height



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Sort-Merge Join

Join computation becomes particularly simple if both inputs are **sorted** with respect to the join attribute(s).

- The **merge join** essentially **merges** both input tables, much like we did for sorting. Both tables are **read once, in parallel**.
- In contrast to sorting, however, we need to be careful whenever a tuple has **multiple** matches in the other relation:

Multiple matches per tuple (disrupts sequential access)

A	B		C	D
"foo"	1	\bowtie B=C	1	false
"foo"	2		2	true
"bar"	2		2	false
"baz"	2		3	true
"baf"	4			

- Merge join is typically used for **equi-joins only**.



Merge Join: Algorithm

Merge join algorithm

```
1 Function: merge_join ( $R, S, \alpha = \beta$ )           //  $\alpha, \beta$ : join col.s  
   in  $R, S$   
2  $r \leftarrow$  position of first tuple in  $R$ ;      //  $r, s, s'$ : cursors over  $R,$   
    $S, S$   
3  $s \leftarrow$  position of first tuple in  $S$  ;  
4 while  $r \neq \langle \text{EOF} \rangle$  and  $s \neq \langle \text{EOF} \rangle$  do //  $\langle \text{EOF} \rangle$ : end of file marker  
5     while  $r.\alpha < s.\beta$  do  
6         | advance  $r$  ;  
7     while  $r.\alpha > s.\beta$  do  
8         | advance  $s$  ;  
9      $s' \leftarrow s$  ;                          // Remember current position in  $S$   
10    while  $r.\alpha = s'.\beta$  do                 // All  $R$ -tuples with same  $\alpha$  value  
11        |  $s \leftarrow s'$  ;                      // Rewind  $s$  to  $s'$   
12        | while  $r.\alpha = s.\beta$  do // All  $S$ -tuples with same  $\beta$  value  
13            | append  $\langle r, s \rangle$  to result ;  
14            | advance  $s$  ;  
15        | advance  $r$  ;
```



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Merge Join: I/O Behavior

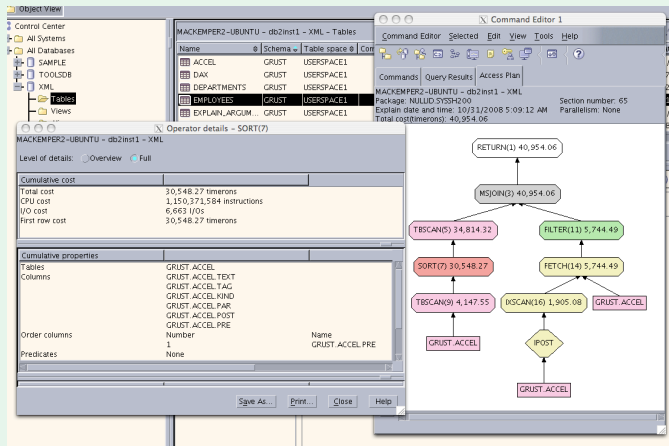
- If both inputs are already sorted **and** there are no exceptionally long sequences of identical key values, the I/O cost of a merge join is $N_R + N_S$ (which is optimal).
- By using **blocked I/O**, these I/O operations can be done almost entirely as **sequential** reads.
- Sometimes, it pays off to explicitly **sort** a (unsorted) relation first, then apply merge join. This is particularly the case if a sorted **output** is beneficial later in the execution plan.
- The final sort pass can also be combined with merge join, avoiding one round-trip to disk and back.

What is the worst-case behavior of merge join?

If both join attributes are constants and carry the same value (*i.e.*, the result is the Cartesian product), merge join degenerates into a nested loops join.

Merge Join: IBM DB2 Plan

DB2. Merge join (left input: sort, right input: sorted index scan)



- **Note:** The FILTER(11) implements the join predicate of the MSJOIN(3).



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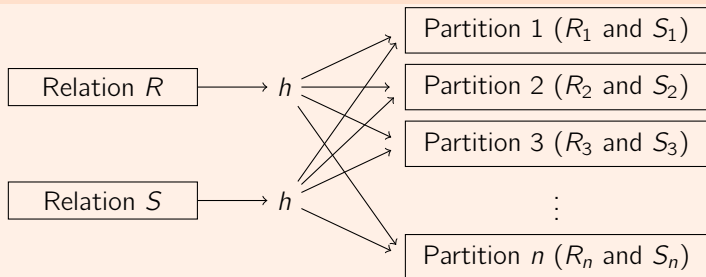
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Hash Join

- Sorting effectively brought related tuples into **spatial proximity**, which we exploited in the merge join algorithm.
- We can achieve a similar effect with **hashing**, too.
- Partition R and S into partitions R_1, \dots, R_n and S_1, \dots, S_n using the **same** hash function (applied to the join attributes).

Hash partitioning for both inputs



- Observe that $R_i \bowtie S_j = \emptyset$ for all $i \neq j$.



Hash Join

- By partitioning the data, we reduced the problem of joining to **smaller sub-relations** R_i and S_i .
- Matching tuples are guaranteed to end up together in the same partition (again: works for equality predicates only).
- We only need to compute $R_i \bowtie S_i$ (for all i).
- By choosing n properly (*i.e.*, the hash function h), partitions become small enough to implement the $R_i \bowtie S_i$ as **in-memory joins**.
- The in-memory join is typically accelerated using a hash table, too. We already did this for the block nested loops join (↗ slide 34).

Intra-partition join via hash table

Use a **different** hash function $h' \neq h$ for the intra-partition join.
Why?



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Hash Join Algorithm



Hash join

```
1 Function: hash_join( $R, S, \alpha = \beta$ )
   /* Partitioning phase */
2 foreach record  $r \in R$  do
3   | append  $r$  to partition  $R_{h(r.\alpha)}$ 
4 foreach record  $s \in S$  do
5   | append  $s$  to partition  $S_{h(s.\beta)}$ 
   /* Intra-partition join phase */
6 foreach partition  $i \in 1, \dots, n$  do
7   | build hash table  $H$  for  $R_i$ , using hash function  $h'$ ;
8   | foreach block  $b \in S_i$  do
9     | foreach record  $s \in b$  do
10    | | probe  $H$  via  $h'(s.\beta)$  and append matching tuples to
       | | result ;
```

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Hash Join—Buffer Requirements

- We assumed that we can create the necessary n partitions in one pass (note that we want $N_{R_i} < (B - 1)$).
- This works out if R consists of **at most** $\approx (B - 1)^2$ pages.

 Why $(B - 1)^2$? Why \approx ?

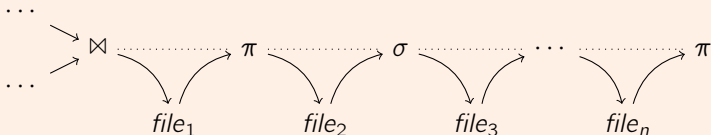
- We can write out at most $B - 1$ partitions in one pass; the R part of each partition should be at most $B - 1$ pages in size.
- Hashing does not guarantee an even distribution. Since the actual size of each partition varies, R must actually be smaller than $(B - 1)^2$.

- Larger input tables require **multiple passes** for partitioning (recursive partitioning).

Orchestrating Operator Evaluation

So far we have assumed that all database operators consume and produce **files** (*i.e.*, on-disk items):

File-based operator input and output



- Obviously, using **secondary storage as the communication channel** causes **a lot of disk I/O**.
- In addition, we suffer from **long response times**:
 - An operator cannot start computing its result before **all** its input files are fully generated (“**materialized**”).
 - Effectively, all operators are executed **in sequence**.



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Unix: Temporary Result Files

- Architecting the query processor in this fashion bears much resemblance with using the Unix shell like this:

File-based Unix command sequencing

```
1 # report "large" XML files below current working dir
2 $ find . -size +500k      > tmp1
3 $ xargs file              < tmp1 > tmp2
4 $ grep -i XML             < tmp2 > tmp3
5 $ cut -d: -f1             < tmp3
6 <output generated here>
7
8 # remove temporary files
9 $ rm -f tmp[0-9]*
```



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Pipelined Evaluation

- Alternatively, each operator could pass its result **directly** on to the next operator (without persisting it to disk first).
 - ⇒ Do not wait until entire file is created, but propagate output **immediately**.
 - ⇒ Start computing results **as early as possible**, *i.e.*, as soon as enough input data is available to start producing output.
- This idea is referred to as **pipelining**.
- The **granularity** in which data is passed may influence performance:
 - Smaller chunks reduce the **response time** of the system.
 - Larger chunks may improve the effectiveness of **(instruction) caches**.
 - Actual systems typically operate **tuple at a time**.



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Unix: Pipelines of Processes

Unix provides a similar mechanism to communicate between processes (“operators”):

Pipeline-based Unix command sequencing

```
1 $ find . -size +500k | xargs file | grep -i XML | cut -d: -f1
2 <output generated here>
```

Execution of this pipe is driven by the **rightmost** operator—all operators act in **parallel**:



- To produce a line of output, `cut` only needs to see the next line of its input: `grep` is requested to produce this input.
- To produce a line of output, `grep` needs to request as many input lines from the `xargs` process until it receives a line containing the string "XML".
- ⋮
- Each line produced by the `find` process is passed through the pipe until it reaches the `cut` process and eventually is echoed to the terminal.



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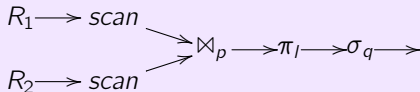
The Volcano Iterator Model

- The **calling interface** used in database execution runtimes is very similar to the one used in Unix process pipelines.
- In databases, this interface is referred to as **open-next-close interface** or **Volcano iterator model**.
- Each operator implements the functions
 - open ()** **Initialize** the operator's internal state.
 - next ()** Produce and return the **next result tuple** or $\langle \text{EOF} \rangle$.
 - close ()** **Clean up** all allocated resources (typically after all tuples have been processed).
- All **state** is kept inside each operator instance:
 - Operators are required to produce a tuple via **next ()**, then **pause**, and later **resume** on a subsequent **next ()** call.

↗ Goetz Graefe. Volcano—An Extensible and Parallel Query Evaluation System. *Trans. Knowl. Data Eng.* vol. 6, no. 1, February 1994.

Volcano-Style Pipelined Evaluation

Example (Pipelined query plan)



- Given a query plan like the one shown above, query evaluation is driven by the query processor like this (just like in the Unix shell):

- 1 The whole plan is initially reset by calling `open ()` on the **root operator**, *i.e.*, `σq.open ()`.
- 2 The `open ()` call is **forwarded** through the plan by the operators themselves (see `σ.open ()` on slide 53).
- 3 Control returns to the query processor.
- 4 The root is requested to produce its next result record, *i.e.*, the call `σq.next ()` is made.
- 5 Operators forward the `next ()` request as needed. **As soon as the next result record is produced, control returns** to the query processor again.



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Volcano-Style Pipelined Evaluation

- In a nutshell, the query processor uses the following routine to evaluate a query plan:

Query plan evaluation driver

```
1 Function: eval (q)
2 q.open ();
3 r ← q.next ();
4 while r ≠ ⟨EOF⟩ do
   | /* deliver record r (print, ship to DB client) */
   | emit (r);
   | r ← q.next ();
   /* resource deallocation now */
7 q.close ();
```



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Volcano-Style Selection (σ_p)

- Input operator (sub-plan root) R , predicate p :

Volcano-style interface for $\sigma_p(R)$

```
1 Function: open ()
```

```
2  $R$ .open () ;
```

```
1 Function: close ()
```

```
2  $R$ .close () ;
```

```
1 Function: next ()
```

```
2 while ( $(r \leftarrow R$ .next ())  $\neq$   $\langle$ EOF $\rangle$ ) do
```

```
3   | if  $p(r)$  then
```

```
4   |   | return  $r$  ;
```

```
5 return  $\langle$ EOF $\rangle$ ;
```



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Volcano-Style Nested Loops Join (\bowtie_p)

A Volcano-style implementation of nested loops join $R \bowtie_p S$?

```
1 Function: open ()
```

```
2  $R.open () ;$ 
```

```
3  $S.open () ;$ 
```

```
4  $r \leftarrow R.next () ;$ 
```

```
1 Function: close ()
```

```
2  $R.close () ;$ 
```

```
3  $S.close () ;$ 
```

```
1 Function: next ()
```

```
2 while ( $r \neq \langle EOF \rangle$ ) do
```

```
3   while ( $(s \leftarrow S.next ()) \neq \langle EOF \rangle$ ) do
```

```
4     if  $p(r, s)$  then
```

```
        $/*$  emit concatenated result  $*/$ 
```

```
5     return  $\langle r, s \rangle ;$ 
```

```
        $/*$  reset inner join input  $*/$ 
```

```
6      $S.close () ;$ 
```

```
7      $S.open () ;$ 
```

```
8      $r \leftarrow R.next () ;$ 
```

```
9 return  $\langle EOF \rangle ;$ 
```



Pipelining (Real DBMS Product)

Volcano-style pipelined selection operator (C code)

```
1 /* eFLTR -- apply filter predicate pred to stream
2    Filter the in-bound stream, only stream elements that fulfill
3    e->pred contribute to the result.  No index support. */
4
5 eRC eOp_FLTR(eOp *ip) {
6     eObj_FLTR *e = (eObj_FLTR *)eObj(ip);
7
8     while (eIntp(e->in) != eEOS) {
9         eIntp(e->pred);
10        if (eT_as_bool(eVal(e->pred))) {
11            eVal(ip) = eVal(e->in);
12            return eOK;
13        }
14    }
15    return eEOS;
16 }
17
18 eRC eOp_FLTR_RST(eOp *ip) {
19     eObj_FLTR *e = (eObj_FLTR *)eObj(ip);
20
21     eReset(e->in);
22     eReset(e->pred);
23     return eOK;
24 }
```



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Blocking Operators

- Pipelining reduces memory requirements and response time since each chunk of input is propagated to the output **immediately**.
- Some operators **cannot** be implemented in such a way.

Which operators do not permit pipelined evaluation?

- (external) sorting (this is also true for Unix `sort`)
 - hash join
 - grouping and duplicate elimination over unsorted input
-
- Such operators are said to be **blocking**.
 - Blocking operators **consume their entire input before they can produce any output**.
 - The data is typically buffered (“materialized”) on disk.



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