Chapter 6

Hash-Based Indexing

Efficient Support for Equality Search

Architecture and Implementation of Database Systems
Summer 2014

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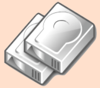

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Linear Hashing
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Hash-Based Indexing

- We now turn to a different family of index structures: **hash indexes.**
- Hash indexes are "unbeatable" when it comes to support for **equality selections**:

Equality selection

```
SELECT *
FROM   R
WHERE  A = k
```

- Further, other query operations internally generate a flood of equality tests (*e.g.*, nested-loop join).
(Non-)presence of hash index support can make a real difference in such scenarios.

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Hashing *vs.* B$^+$-trees

- Hash indexes provide **no support for range queries**, however (hash indexes are also know as **scatter storage**).
- In a B$^+$-tree-world, to locate a record with key k means to **compare** k with other keys k' organized in a (tree-shaped) search data structure.
- Hash indexes **use the bits of** k **itself** (independent of all other stored records) to find the location of the associated record.
- We will now briefly look into **static hashing** to illustrate the basics.
    - Static hashing does *not* handle updates well (much like ISAM).
    - Later, we introduce **extendible hashing** and **linear hashing** which refine the hashing principle and adapt well to record insertions and deletions.

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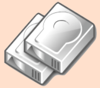

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Static Hashing

- To build a **static hash index** on attribute `A`:

Build static hash index on column `A`

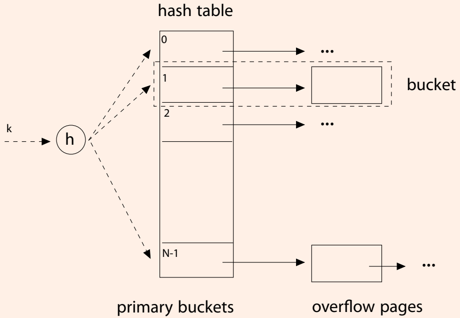
1. Allocate a fixed area of N (successive) disk pages, the so-called **primary buckets**.
2. In each bucket, install a pointer to a chain of **overflow pages** (initially set the pointer to **null**).
3. Define a **hash function** h with *range* $[0, \ldots, N-1]$. The *domain* of h is the type of `A`, *e.g.*.

$$h : \mathtt{INTEGER} \to [0, \ldots, N-1]$$

if `A` is of SQL type `INTEGER`.

Static Hashing

Static hash table



- A primary bucket and its associated chain of overflow pages is referred to as a **bucket** (above).
- Each bucket contains **index entries** $k*$ (implemented using any of the variants A, B, C, see slide 2.22.

Static Hashing

- To perform $\texttt{hsearch}(k)$ (or $\texttt{hinsert}(k)/\texttt{hdelete}(k)$) for a record with key $\texttt{A} = k$:

Static hashing scheme

1. **Apply hash function** h to the key value, *i.e.*, compute $h(k)$.
2. **Access the primary bucket page** with number $h(k)$.
3. Search (insert/delete) subject record on this page or, if required, **access the overflow chain** of bucket $h(k)$.

- If the hashing scheme works well and overflow chain access is avoidable,
    - $\texttt{hsearch}(k)$ requires a **single I/O operation**,
    - $\texttt{hinsert}(k)/\texttt{hdelete}(k)$ require **two I/O operations**.

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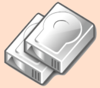
Static Hashing: Collisions and Overflow Chains

- At least for static hashing, **overflow chain management** is important.
- Generally, we do **not** want hash function h to avoid **collisions**, *i.e.*,

$$h(k) = h(k') \qquad \text{even if} \qquad k \neq k'$$

  (otherwise we would need as many primary bucket pages as different key values in the data file).
- At the same time, we want h to **scatter** the key attribute domain **evenly** across $[0, \ldots, N-1]$ to avoid the development of long overflow chains for few buckets. This makes the hash tables' I/O behavior non-uniform and unpredictable for a query optimizer.

- Such "good" hash functions are hard to discover, unfortunately.



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The Birthday Paradox (Need for Overflow Chain Management)

Example (The birthday paradox)

Consider the people in a group as the **domain** and use their birthday as **hash function** h ($h : Person \rightarrow [0, \ldots, 364]$).

> *If the group has 23 or more members, chances are $> 50\,\%$ that two people share the same birthday (**collision**).*

Check: Compute the probability that n people *all have different birthdays*:

Function: different_birthday (n)

if $n = 1$ **then**
    return 1;
else
    return $\underbrace{\texttt{different\_birthday}(n-1)}_{\substack{\text{probability that } n-1 \text{ persons}\\ \text{have different birthdays}}} \times \underbrace{\frac{365 - (n-1)}{365}}_{\substack{\text{probability that } n\text{th person}\\ \text{has birthday different from}\\ \text{first } n-1 \text{ persons}}}$;

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Hash Functions

- It is impossible to generate truly random hash values from the non-random key values found in actual table. Can we define hash functions that scatter even better than a random function?

Hash function

1. **By division**. Simply define

$$h(k) = k \bmod N \enspace .$$

This guarantees the range of $h(k)$ to be $[0, \ldots, N-1]$.

Note: Choosing $N = 2^d$ for some d effectively considers the least d bits of k only. **Prime numbers** work best for N.

2. **By multiplication**. Extract the fractional part of $Z \cdot k$ (for a specific Z[1]) and multiply by arbitrary hash table size N:

$$h(k) = \left\lfloor N \cdot \left(Z \cdot k - \lfloor Z \cdot k \rfloor\right) \right\rfloor$$

[1] The (inverse) **golden ratio** $Z = 2/(\sqrt{5}+1) \approx 0.6180339887$ is a good choice. See D.E.Knuth, "*Sorting and Searching*."

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Static Hashing and Dynamic Files

- For a static hashing scheme:
  - If the underlying **data file grows**, the development of overflow chains spoils the otherwise predictable behavior hash I/O behavior (1–2 I/O operations).
  - If the underlying **data file shrinks**, a significant fraction of primary hash buckets may be (almost) empty—a waste of page space.
- As in the ISAM case, however, static hashing has advantages when it comes to concurrent access.
- We may perodicially **rehash** the data file to restore the ideal situation (20 % free space, no overflow chains).

  $\Rightarrow$ Expensive and the index cannot be used while rehashing is in progress.

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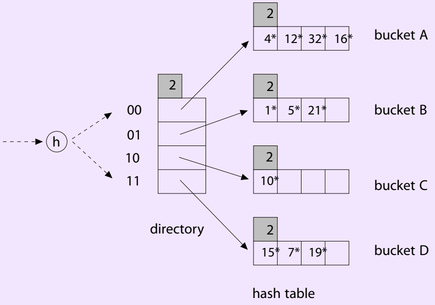
Extendible Hashing

- **Extendible Hashing** can adapt to growing (or shrinking) data files.
- To keep track of the actual primary buckets that are part of the current hash table, we hash via an **in-memory bucket directory**:

Example (Extendible hash table setup; ignore the $\boxed{2}$ fields for now[2])



[2] **Note**: This figure depicts the entries as $h(k)*$, not $k*$.

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Extendible Hashing: Search

Search for a record with key k

1. Apply h, *i.e.*, compute $h(k)$.
2. Consider the last $\boxed{2}$ bits of $h(k)$ and follow the corresponding directory pointer to find the bucket.

Example (Search for a record)

To find a record with key k such that $h(k) = 5 = 101_2$, follow the second directory pointer ($101_2 \wedge 11_2 = 01_2$) to bucket B, then use entry $5*$ to access the wanted record.

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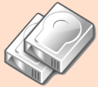
Extendible Hashing: Global and Local Depth

Global and local depth annotations

- **Global depth** ($\boxed{n}$ at hash directory):
  Use the last n bits of $h(k)$ to lookup a bucket pointer in the directory (the directory size is 2^n).



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Extendible Hashing: Global and Local Depth

Global and local depth annotations

- **Global depth** ($\boxed{n}$ at hash directory):
  Use the last n bits of $h(k)$ to lookup a bucket pointer in the directory (the directory size is 2^n).

- **Local depth** ($\boxed{d}$ at individual buckets):
  The hash values $h(k)$ of all entries in this bucket agree on their last d bits.

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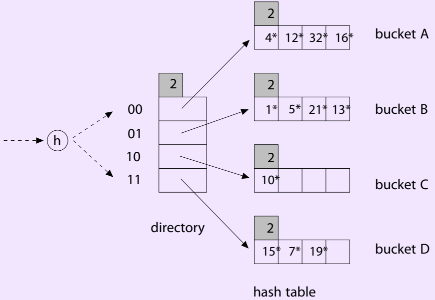
Extendible Hashing: Insert

Insert record with key k

1. Apply h, *i.e.*, compute $h(k)$.
2. Use the last $\boxed{n}$ bits of $h(k)$ to lookup the bucket pointer in the directory.
3. If the *primary bucket* still has capacity, store $h(k)*$ in it. (Otherwise …?)

Example (Insert record with $h(k) = 13 = 11\underline{01}_2$)



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Extendible Hashing: Insert, Bucket Split

Example (Insert record with $h(k) = 20 = 101\underline{00}_2$)

Insertion of a record with $h(k) = 20 = 101\underline{00}_2$ leads to **overflow in primary bucket** A. Initiate a **bucket split** for A.

1. **Split** bucket A (creating a new bucket A2) and use bit position $\boxed{d} + 1$ to redistribute the entries:

$$
\begin{aligned}
4 &= \underline{1}00_2 \\
12 &= 1\underline{1}00_2 \\
32 &= 100\underline{0}00_2 \\
16 &= 10\underline{0}00_2 \\
20 &= 10\underline{1}00_2
\end{aligned}
$$

0 → Bucket A: | 32 | 16 |

1 → Bucket A2: | 4 | 12 | 20 |

Note: We now need $\boxed{3}$ bits to discriminate between the old bucket A and the new split bucket A2.

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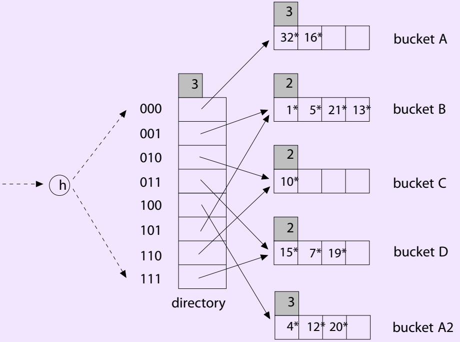
Extendible Hashing: Insert, Directory Doubling

Example (Insert record with $h(k) = 20 = 101\underline{00}_2$)

2. *In the present case*, we need to **double the directory** by simply copying its original pages (we now use $\boxed{2} + 1 = \boxed{3}$ bits to lookup a bucket pointer).
3. Let bucket pointer for $\underline{1}00_2$ point to A2 (the directory pointer for $\underline{0}00_2$ still points to bucket A):



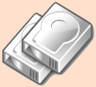
Extendible Hashing: Insert

If we split a bucket with local depth $\boxed{d} < \boxed{n}$ (global depth), directory doubling is *not* necessary:

Example (Insert record with $h(k) = 9 = 1\underline{001}_2$)

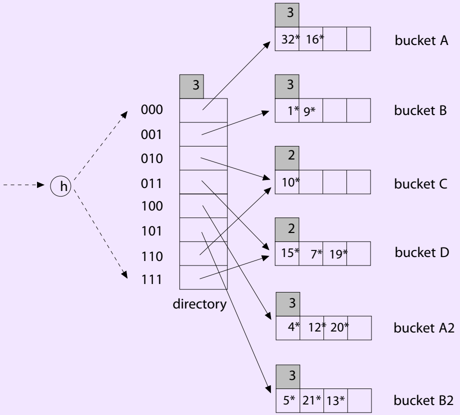
- Insert record with key k such that $h(k) = 9 = 1\underline{001}_2$.
- The associated bucket B is split, creating a new bucket B2. Entries are redistributed. New local depth of B and B2 is $\boxed{3}$ and thus does *not* exceed the global depth of $\boxed{3}$.

$\Rightarrow$ Modifying the directory's bucket pointer for $\underline{101}_2$ is sufficient (see following slide).



Extendible Hashing: Insert

Example (After insertion of record with $h(k) = 9 = \underline{1001}_2$)



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Extendible Hashing: Search Procedure

- The following `hsearch(·)` and `hinsert(·)` procedures operate over an in-memory array representation of the bucket directory $bucket[0, \ldots, 2^{\boxed{n}} - 1]$.

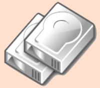
Extendible Hashing: Search

Function: $\texttt{hsearch}(k)$

$n \leftarrow \boxed{n}$; `/* global depth */`

$b \leftarrow h(k)\ \&\ (2^n - 1)$; `/* mask all but the low` n `bits */`

return $bucket[b]$;



Extendible Hashing: Insert Procedure

Extendible Hashing: Insertion

Function: `hinsert(`$k*$`)`

$n \leftarrow \boxed{n}$; `/* global depth */`

$b \leftarrow$ `hsearch(`k`)` ;

if b has capacity **then**

  Place $k*$ in bucket b ;

  return;

`/* overflow in bucket b, need to split */`

$d \leftarrow \boxed{d}_b$; `/* local depth of hash bucket b */`

Create a new empty bucket $b2$;

`/* redistribute entries of b including k* */`

$\vdots$

Extendible Hashing: Insert Procedure (continued)

Extendible Hashing: Insertion (cont'd)

⋮

`/* redistribute entries of` b `including` $k*$ `*/`

foreach $k'*$ in bucket b **do**

  if $h(k')\ \&\ 2^d \neq 0$ **then**

    Move $k'*$ to bucket $b2$;

`/* new local depths for buckets` b `and` $b2$ `*/`

$\boxed{d}_b \leftarrow d+1$;

$\boxed{d}_{b2} \leftarrow d+1$;

if $n < d+1$ **then**

  `/* we need to double the directory */`

  Allocate 2^n new directory entries $bucket[2^n,\ldots,2^{n+1}-1]$;

  Copy $bucket[0,\ldots,2^n-1]$ into $bucket[2^n,\ldots,2^{n+1}-1]$;

  $\boxed{n} \leftarrow n+1$;

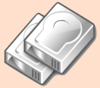
  `/* update the bucket directory to point to` $b2$ `*/`

  $bucket[(h(k)\ \&\ (2^n-1)) \mid 2^n] \leftarrow addr(b2)$

Extendible Hashing: Overflow Chains? / Delete

✎ Overflow chains?

Extendible hashing uses overflow chains hanging off a bucket only as a resort. Under which circumstances will extendible hashing create an overflow chain?



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Extendible Hashing: Overflow Chains? / Delete

✎ Overflow chains?

Extendible hashing uses overflow chains hanging off a bucket only as a resort. Under which circumstances will extendible hashing create an overflow chain?

If considering $d + 1$ bits does *not* lead to satisfying record redistribution in procedure `hinsert(k)` (skewed data, hash collisions).

- Deleting an entry $k*$ from a bucket may leave its bucket completely (or almost) empty.
- Extendible hashing then tries to **merge** the empty bucket and its associated partner bucket.

✎ Extendible hashing: deletion

When is local depth decreased? When is global depth decreased?

(Try to work out the details on your own.)

Linear Hashing

- **Linear hashing** can, just like extendible hashing, adapt its underlying data structure to record insertions and deletions:
  - Linear hashing **does not need a hash directory** in addition to the actual hash table buckets.
  - Linear hashing can define **flexible criteria that determine when a bucket is to be split**,
  - Linear hashing, however, may perform badly if the key distribution in the data file is *skewed*.

- We will now investigate linear hashing in detail and come back to the points above as we go along.

- The core idea behind linear hashing is to use an **ordered family of hash functions**, $h_0, h_1, h_2, \ldots$ (traditionally the subscript is called the hash function's *level*).

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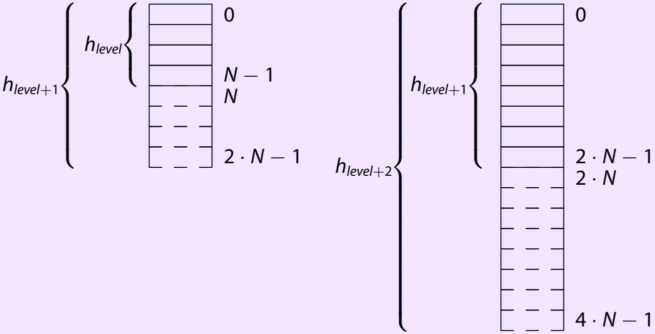
Linear Hashing: Hash Function Family

- We design the family so that the **range of $h_{level+1}$ is twice as large as the range of h_{level}** (for $level = 0, 1, 2, \ldots$).

Example (h_{level} with range $[0, \ldots, N-1]$)



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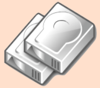
Linear Hashing: Hash Function Family

- Given an initial hash function h and an initial hash table size N, one approach to define such a family of hash functions h_0, h_1, h_2, … would be:

Hash function family

$$h_{level}(k) = h(k) \bmod (2^{level} \cdot N) \qquad (level = 0, 1, 2, \dots)$$



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Linear Hashing: Basic Scheme

Basic linear hashing scheme

1. Initialize: $level \leftarrow 0$, $next \leftarrow 0$.

2. The **current hash function** in use for searches (insertions/deletions) is h_{level}, active hash table buckets are those in h_{level}'s range: $[0, \ldots, 2^{level} \cdot N - 1]$.

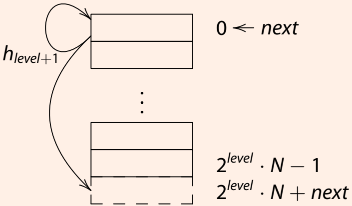
3. **Whenever** we realize that the current **hash table overflows**, *e.g.*,
   - insertions filled a primary bucket beyond c % capacity,
   - or the overflow chain of a bucket grew longer than p pages,
   - or ⟨*insert your criterion here*⟩

   we **split the bucket** at hash table position *next*
   (in general, this is **not the bucket which triggered the split!**)

Linear Hashing: Bucket Split

Linear hashing: bucket split

1. **Allocate a new bucket, append** it to the hash table (its position will be $2^{level} \cdot N + next$).
2. **Redistribute** the entries in bucket *next* by **rehashing** them via $h_{level+1}$ (some entries will remain in bucket *next*, some go to bucket $2^{level} \cdot N + next$). For $next = 0$:



3. **Increment** *next* by 1.

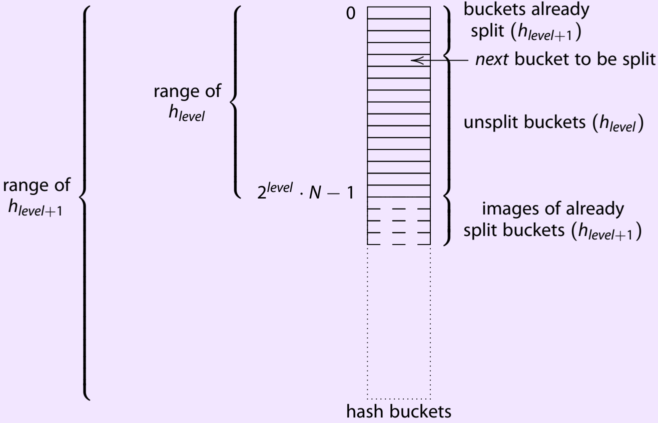
$\Rightarrow$ All buckets with positions $<$ *next* have been rehashed.

Linear Hashing: Rehashing

Searches need to take current *next* position into account

$$h_{level}(k) \begin{cases} < next : \text{we hit an already split bucket, } \textbf{rehash} \\ \geqslant next : \text{we hit a yet unsplit bucket, bucket found} \end{cases}$$

Example (Current state of linear hashing scheme)



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Linear Hashing: Split Rounds

✎ When *next* is incremented beyond hash table size. . . ?

A bucket split increments *next* by 1 to mark the next bucket to be split. How would you propose to handle the situation when *next* is incremented *beyond* the last current hash table position, *i.e.*

$$next > 2^{level} \cdot N - 1?$$

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Linear Hashing: Split Rounds

✎ When *next* is incremented beyond hash table size. . . ?

A bucket split increments *next* by 1 to mark the next bucket to be split. How would you propose to handle the situation when *next* is incremented *beyond* the last current hash table position, *i.e.*

$$next > 2^{level} \cdot N - 1?$$

Answer:

- If $next > 2^{level} \cdot N - 1$, **all buckets** in the current hash table are hashed via function $h_{level+1}$.

⇒ Proceed in a **round-robin fashion**:
If $next > 2^{level} \cdot N - 1$, then

1. increment *level* by 1,
2. $next \leftarrow 0$ (start splitting from hash table top again).

- In general, an overflowing bucket is *not* split immediately, but—due to round-robin splitting—no later than in the following round.

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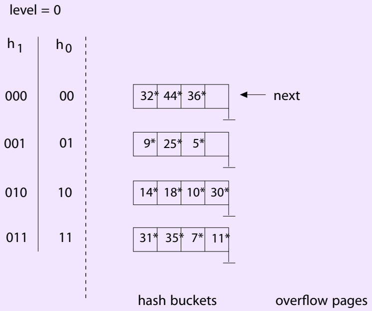
Linear Hashing: Running Example

Linear hash table setup:

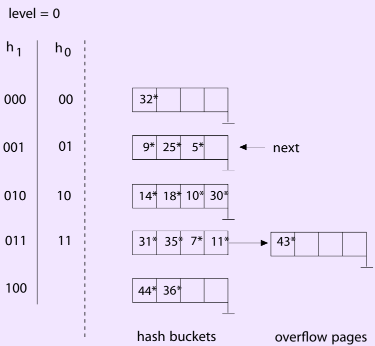
- Bucket capacity of 4 entries, initial hash table size $N = 4$.
- Split criterion: allocation of a page in an overflow chain.

Example (Linear hash table, $h_{level}(k)*$ shown)



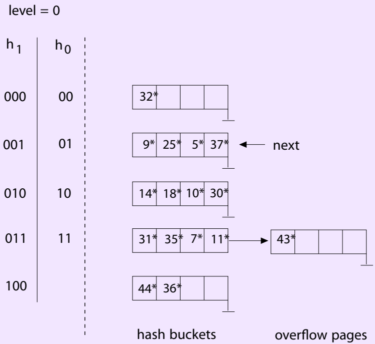
Linear Hashing: Running Example

Example (Insert record with key k such that $h_0(k) = 43 = 101011_2$)



Linear Hashing: Running Example

Example (Insert record with key k such that $h_0(k) = 37 = 100101_2$)



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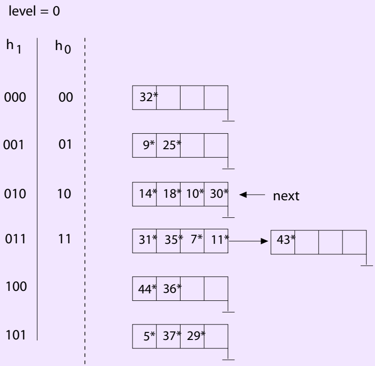
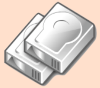
Linear Hashing: Running Example

Example (Insert record with key k such that $h_0(k) = 29 = 11101_2$)





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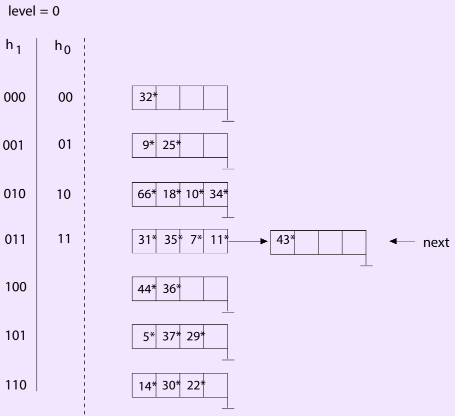
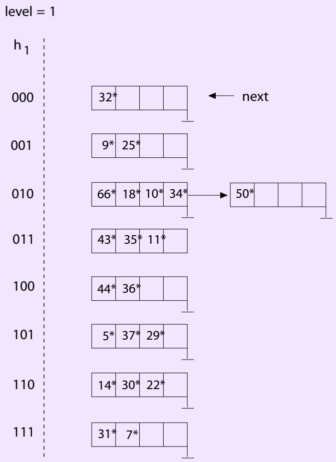
Example (Insert three records with key k such that $h_0(k) = 22 = 10110_2$ / $66 = 1000010_2$ / $34 = 100010_2$)



Linear Hashing: Running Example

Example (Insert record with key k such that $h_0(k) = 50 = 110010_2$)



Rehashing a bucket requires rehashing its overflow chain, too.

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Linear Hashing: Search Procedure

- Procedures operate over hash table bucket (page) address array $bucket[0, \ldots, 2^{level} \cdot N - 1]$.
- Variables *level*, *next* are hash-table globals, *N* is constant.

Linear hashing: search

Function: `hsearch`(k)

$b \leftarrow h_{level}(k)$;

if $b < next$ **then**

    /* b has already been split, record for key k */

    /* may be in bucket b or bucket $2^{level} \cdot N + b$ */

    /* $\Rightarrow$ rehash */

    $b \leftarrow h_{level+1}(k)$;

/* return address of bucket at position b */

return $bucket[b]$;

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Linear Hashing: Insert Procedure

Linear hashing: insert

Function: `hinsert(`$k*$`)`

$b \leftarrow h_{level}(k)$;

if $b < next$ **then**

    `/* rehash */`

    $b \leftarrow h_{level+1}(k)$;

Place $k*$ in $bucket[b]$;

if $overflow(bucket[b])$ **then**

    Allocate new page b' ;

    `/* Grow hash table by one page */`

    $bucket[2^{level} \cdot N + next] \leftarrow addr(b')$;

    $\vdots$

- Predicate $overflow(\cdot)$ is a tunable parameter:
  whenever $overflow(bucket[b])$ returns *true*, trigger a split.

Linear Hashing: Insert Procedure (continued)

Linear hashing: insert (cont'd)

```
⋮
if overflow(···) then
    ⋮
    foreach entry k'* in bucket[next] do
        /* redistribute                                   */
        Place k'* in bucket[h_{level+1}(k')] ;
    next ← next + 1 ;
    /* did we split every bucket in the hash?            */
    if next > 2^{level} · N − 1 then
        /* hash table size doubled, split from top        */
        level ← level + 1 ;
        next ← 0 ;
return;
```


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Linear Hashing: Delete Procedure (Sketch)

- Linear hashing deletion essentially behaves as the "inverse" of `hinsert(·)`:

Linear hashing: delete (sketch)

Function: `hdelete(`k`)`

$b \leftarrow h_{level}(k)$;

$\vdots$

Remove $k*$ from $bucket[b]$;

if $empty(bucket[b])$ **then**

    Move entries from page $bucket[2^{level} \cdot N + next - 1]$

        to page $bucket[next - 1]$;

    $next \leftarrow next - 1$;

    if $next < 0$ **then**

        `/* round-robin scheme for deletion */`

        $level \leftarrow level - 1$;

        $next \leftarrow 2^{level} \cdot N - 1$;

return;

- Possible: replace $empty(\cdot)$ by suitable $underflow(\cdot)$ predicate.

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