

- Two-Way Merge Sort
- External Merge Sort
- Comparisons
- Replacement Sort
- B^+ -trees for Sorting

Chapter 7

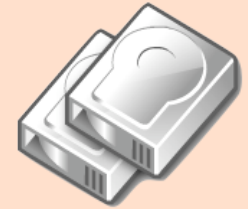
External Sorting

Sorting Tables Larger Than Main Memory

Architecture and Implementation of Database Systems

Winter 2010/11





Two-Way Merge Sort
External Merge Sort
Comparisons
Replacement Sort
B⁺-trees for Sorting

Challenges lurking behind a SQL query

aggregation

```
SELECT C.CUST_ID, C.NAME, SUM(O.TOTAL) AS REVENUE
FROM CUSTOMERS AS C, ORDERS AS O
WHERE C.ZIPCODE BETWEEN 8000 AND 8999
AND C.CUST_ID = O.CUST_ID
GROUP BY C.CUST_ID
ORDER BY C.CUST_ID, C.NAME
```

grouping

selection

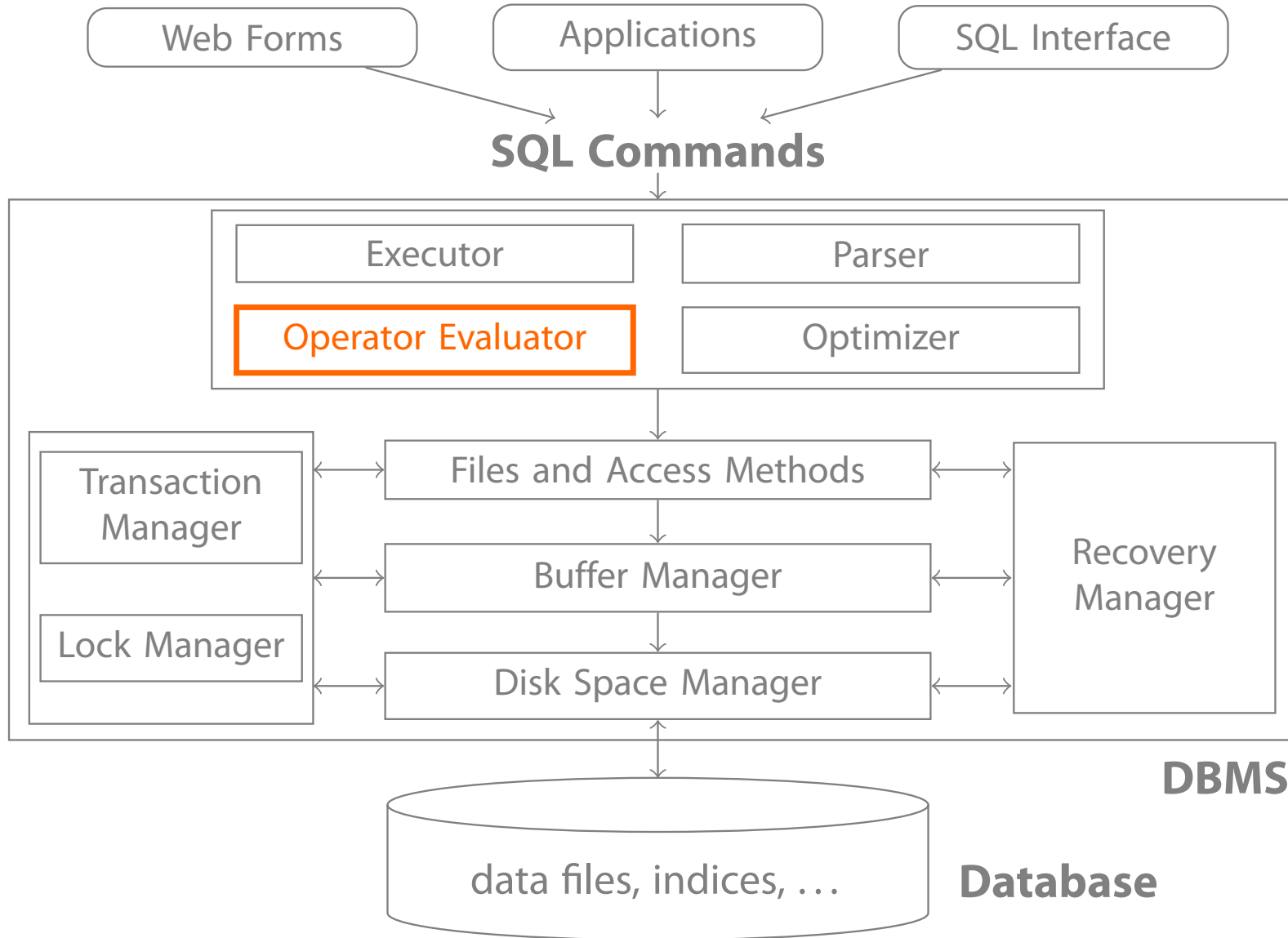
join

sorting

A DBMS **query processor** needs to perform a number of tasks

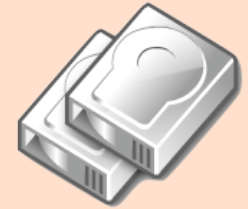
- with **limited memory resources**,
- over **large amounts of data**,
- yet **as fast as possible**.

Query Processing



External Sorting

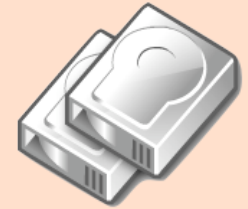
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Query Processing

Sorting

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Query plans and operators

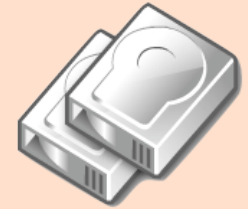
Query Processing

Sorting

Two-Way Merge Sort
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- DBMS does *not* execute a query as a large monolithic block but rather provides a number of specialized routines, the **query operators**.
 - Operators are “plugged together” to form a network of operators, a **plan**, that is capable of evaluating a given query.
 - Each operator is carefully implemented to perform a specific task well (*i.e.*, time- and space-efficient).
-
- **Now:** Zoom in on the details of the implementation of one of the most basic and important operators: **sort**.

Query Processing: Sorting



- **Sorting** stands out as a useful operation, explicit or implicit:

Explicit sorting via the SQL ORDER BY clause

```
1      SELECT  A,B,C
2      FROM    R
3      ORDER BY A
```

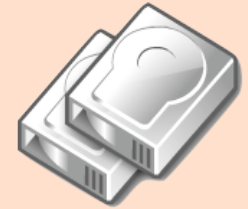
Implicit sorting, e.g., for duplicate elimination

```
1      SELECT DISTINCT A,B,C
2      FROM            R
```

Implicit sorting, e.g., to prepare equi-join

```
1      SELECT R.A,S.Y
2      FROM  R,S
3      WHERE R.B = S.X
```

- Further:
Grouping via GROUP BY, B⁺-tree bulk loading, sorted *rid* scans after access to unclustered indexes, ...



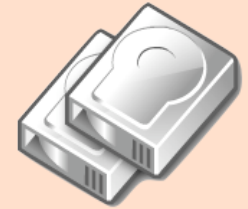
Sorting

- A file is **sorted** with respect to **sort key** k and **ordering** θ , if for any two records $r_{1,2}$ with r_1 preceding r_2 in the file, we have that their corresponding keys are in θ -order:

$$r_1 \theta r_2 \quad \Leftrightarrow \quad r_1.k \theta r_2.k \ .$$

- A key may be a single attribute as well as an ordered list of attributes. In the latter case, order is defined **lexicographically**. Consider: $k = (A, B)$, $\theta = <$:

$$r_1 < r_2 \quad \Leftrightarrow \quad r_1.A < r_2.A \vee \\ (r_1.A = r_2.A \wedge r_1.B < r_2.B) \ .$$



- As it is a principal goal not to restrict the file sizes a DBMS can handle, we face a fundamental problem:

*How can we sort a file of records whose **size exceeds the available main memory space** (let alone the available buffer manager space) by far?*

- Approach the task in a two-phase fashion:
 - ① Sorting a file of arbitrary size is possible even if **three pages** of buffer space is all that is available.
 - ② Refine this algorithm to make effective use of larger and thus more realistic buffer sizes.
- As we go along, consider a number of further optimizations in order to **reduce the overall number of required page I/O operations**.

Two-Way Merge Sort

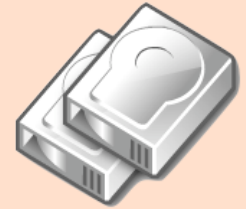
We start with **two-way merge sort**, which can sort files of arbitrary size with only **three pages** of buffer space.

Two-way merge sort

Two-way merge sort sorts a file with $N = 2^k$ pages in multiple **passes**, each of them producing a certain number of sorted sub-files called **runs**.

- **Pass 0** sorts each of the 2^k input pages individually and in **main memory**, resulting in 2^k sorted runs.
- **Subsequent passes merge** pairs of runs into larger runs. Pass n produces 2^{k-n} runs.
- **Pass k** leaves only one run left, the sorted overall result.

During each pass, we consult every page in the file. Hence, $k \cdot N$ page reads and $k \cdot N$ page writes are required to sort the file.



Basic Two-Way Merge Sort Idea

Pass 0 (Input: $N = 2^k$ unsorted pages; Output: 2^k sorted runs)

1. **Read** N pages, **one page at a time**
2. **Sort** records, page-wise, in main memory.
3. **Write** sorted pages to disk (each page results in a **run**).

This pass requires **one page** of buffer space.

Pass 1 (Input: $N = 2^k$ sorted runs; Output: 2^{k-1} sorted runs)

1. Open two runs r_1 and r_2 from Pass 0 for reading.
2. **Merge** records from r_1 and r_2 , reading input page-by-page.
3. **Write** new two-page run to disk (page-by-page).

This pass requires **three pages** of buffer space.

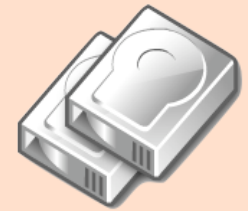
⋮

Pass n (Input: 2^{k-n+1} sorted runs; Output: 2^{k-n} sorted runs)

1. Open two runs r_1 and r_2 from Pass $n - 1$ for reading.
2. **Merge** records from r_1 and r_2 , reading input page-by-page.
3. **Write** new 2^n -page run to disk (page-by-page).

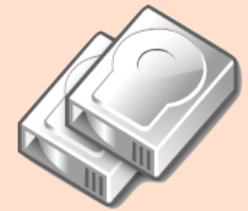
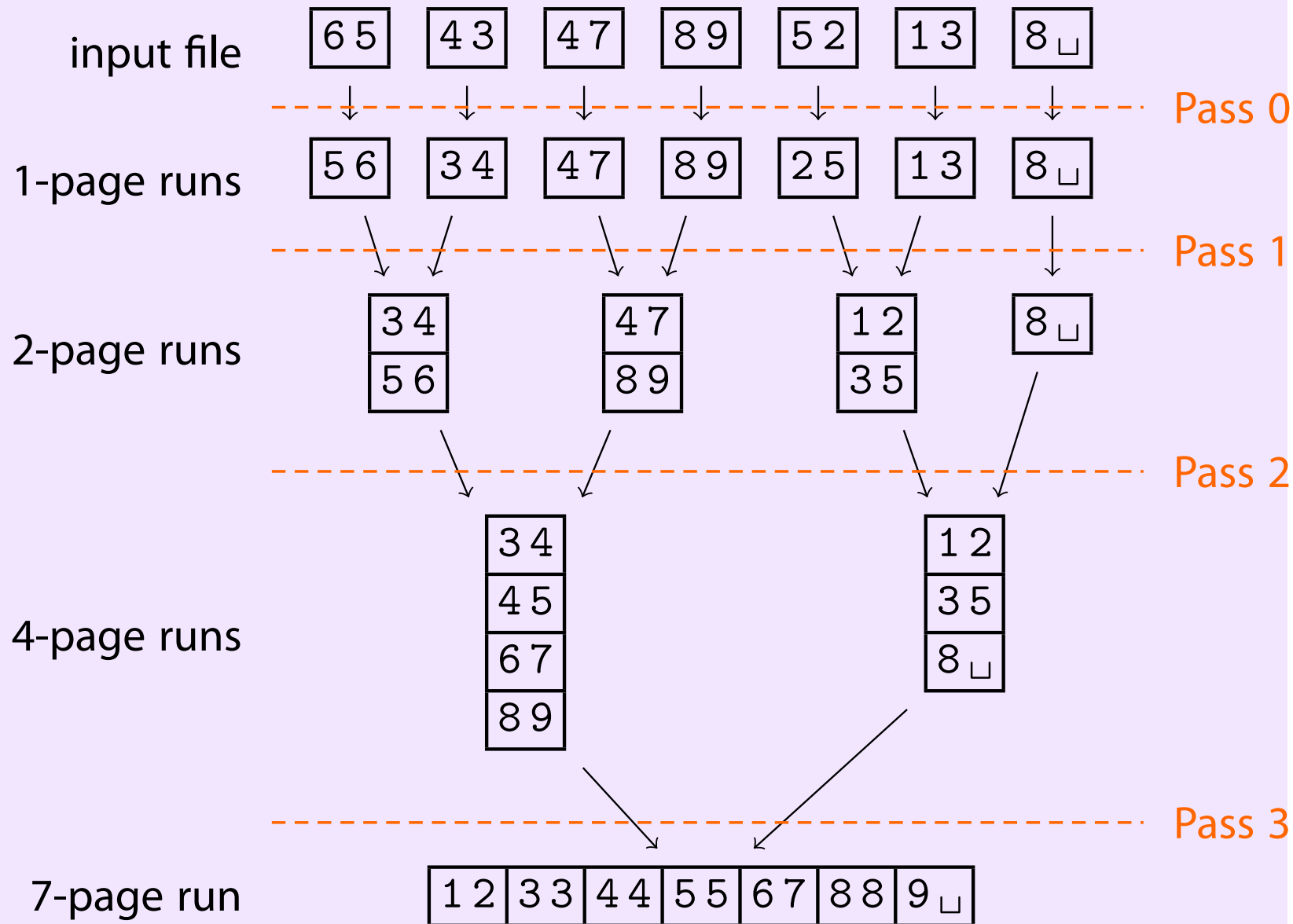
This pass requires **three pages** of buffer space.

⋮

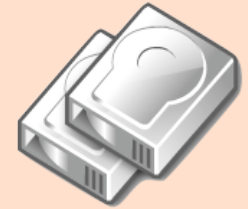


Two-way Merge Sort: Example

Example (7-page file, two records per page, keys k shown, $\theta = \langle \rangle$)



Two-Way Merge Sort: Algorithm

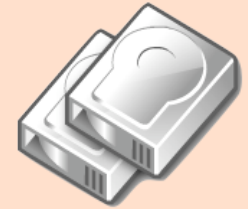


Two-way merge sort, $N = 2^k$

```
1 Function: two_way_merge_sort (file, N)
  /* Pass 0: create N sorted single-page runs
    (in-memory sort) */
2 foreach page p in file do
3   [ read p into memory, sort it, write it out into a new run;
  /* next k passes merge pairs of runs, until only one
    run is left */
4 for n in 1...k do
5   [ for r in 0... $2^{k-n} - 1$  do
6     [ merge runs  $2 \cdot r$  and  $2 \cdot r + 1$  from previous pass into a
7       [ new run, reading the input runs one page at a time;
8         [ delete input runs  $2 \cdot r$  and  $2 \cdot r + 1$  ;
  result  $\leftarrow$  last output run;
```

Each merge requires **three buffer frames** (two to read the two input files and one to construct output pages).

Two-Way Merge Sort: I/O Behavior



- To sort a file of N pages, in each pass we read N pages, sort/merge, and write N pages out again:

$$2 \cdot N \quad \text{I/O operations per pass}$$

- Number of passes:

$$\underbrace{1}_{\text{Pass 0}} + \underbrace{\lceil \log_2 N \rceil}_{\text{Passes 1, \dots, } k}$$

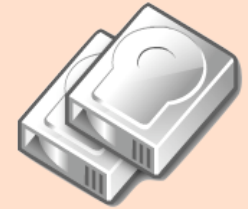
- **Total number of I/O operations:**

$$2 \cdot N \cdot (1 + \lceil \log_2 N \rceil)$$

 **How many I/Os does it take to sort an 8 GB file?**

Assume a page size of 8 KB (with 1000 records each).

External Merge Sort

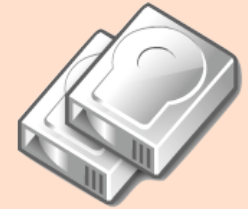


- So far we have “voluntarily” used only three pages of buffer space.

*How could we **make effective use of a significantly larger buffer page pool** (of, say, B frames)?*

- Basically, there are two knobs we can turn and tune:
 - 1 **Reduce the number of initial runs** by using the full buffer space during the in-memory sort.

External Merge Sort



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*How could we **make effective use of a significantly larger buffer page pool** (of, say, B frames)?*

- Basically, there are two knobs we can turn and tune:
 - 1 **Reduce the number of initial runs** by using the full buffer space during the in-memory sort.
 - 2 **Reduce the number of passes** by merging more than two runs at a time.

Reducing the Number of Initial Runs

With B frames available in the buffer pool, **we can read B pages at a time during Pass 0** and sort them in memory (↗ slide 9):

Pass 0 (Input: N unsorted pages; Output: $\lceil N/B \rceil$ sorted runs)

1. **Read** N pages, B pages at a time
 2. **Sort** records in main memory.
 3. **Write** sorted pages to disk (resulting in $\lceil N/B \rceil$ runs).
- This pass uses B pages of buffer space.

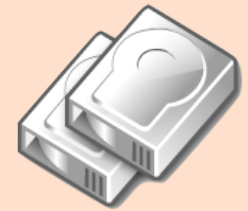
The **number of initial runs** determines the **number of passes** we need to make (↗ slide 12):

⇒ **Total number of I/O operations:**

$$2 \cdot N \cdot (1 + \lceil \log_2 \lceil N/B \rceil \rceil) .$$

 **How many I/Os does it take to sort an 8 GB file now?**

Again, assume 8 KB pages. Available buffer space is $B = 1,000$.



Reducing the Number of Passes

With B frames available in the buffer pool, we can **merge $B - 1$ pages at a time** (leaving one frame as a write buffer).

Pass n (Input: $\frac{\lceil N/B \rceil}{(B-1)^{n-1}}$ sorted runs; Output: $\frac{\lceil N/B \rceil}{(B-1)^n}$ sorted runs)

1. Open $B - 1$ runs $r_1 \dots r_{B-1}$ from Pass $n - 1$ for reading.
2. **Merge** records from $r_1 \dots r_{B-1}$, reading page-by-page.
3. **Write** new $B \cdot (B - 1)^{n-1}$ -page run to disk (page-by-page).

This pass requires B **pages** of buffer space.

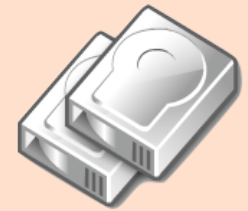
With B pages of buffer space, we can do a $(B - 1)$ -**way merge**.

⇒ **Total number of I/O operations:**

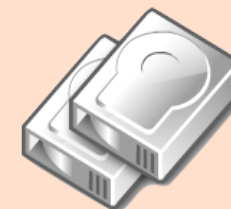
$$2 \cdot N \cdot \left(1 + \lceil \log_{B-1} \lceil N/B \rceil \rceil\right) .$$

 **How many I/Os does it take to sort an 8 GB file now?**

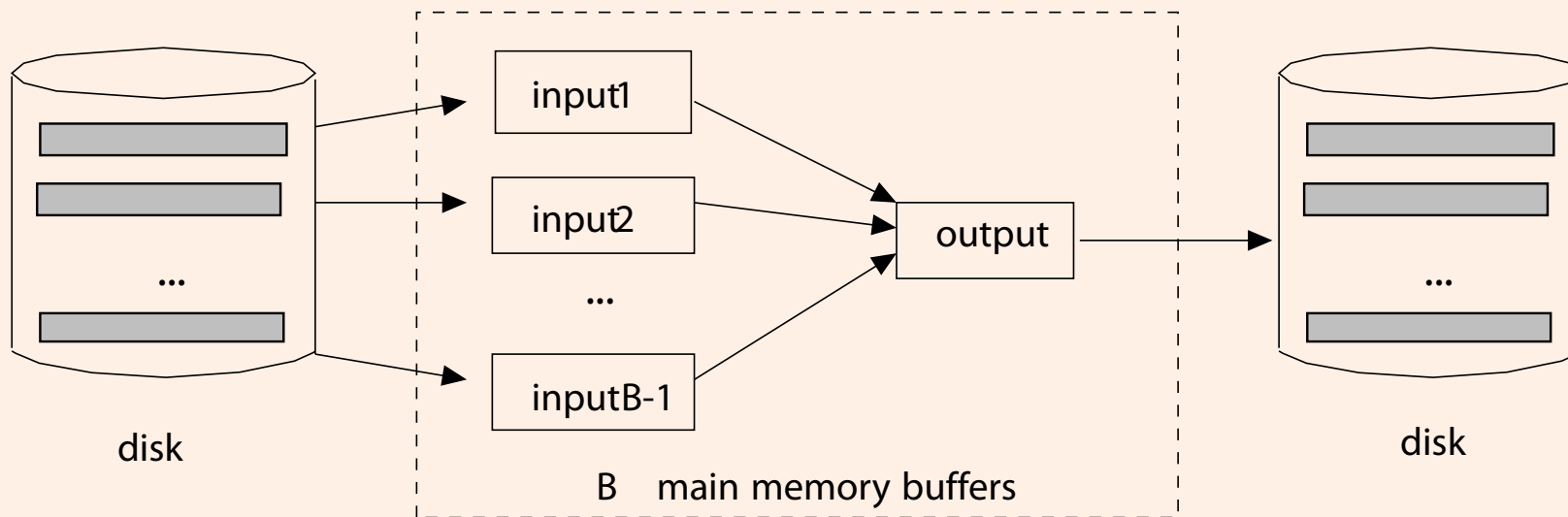
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Reducing the Number of Passes



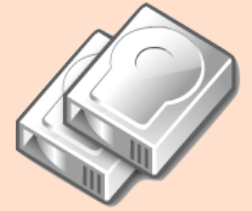
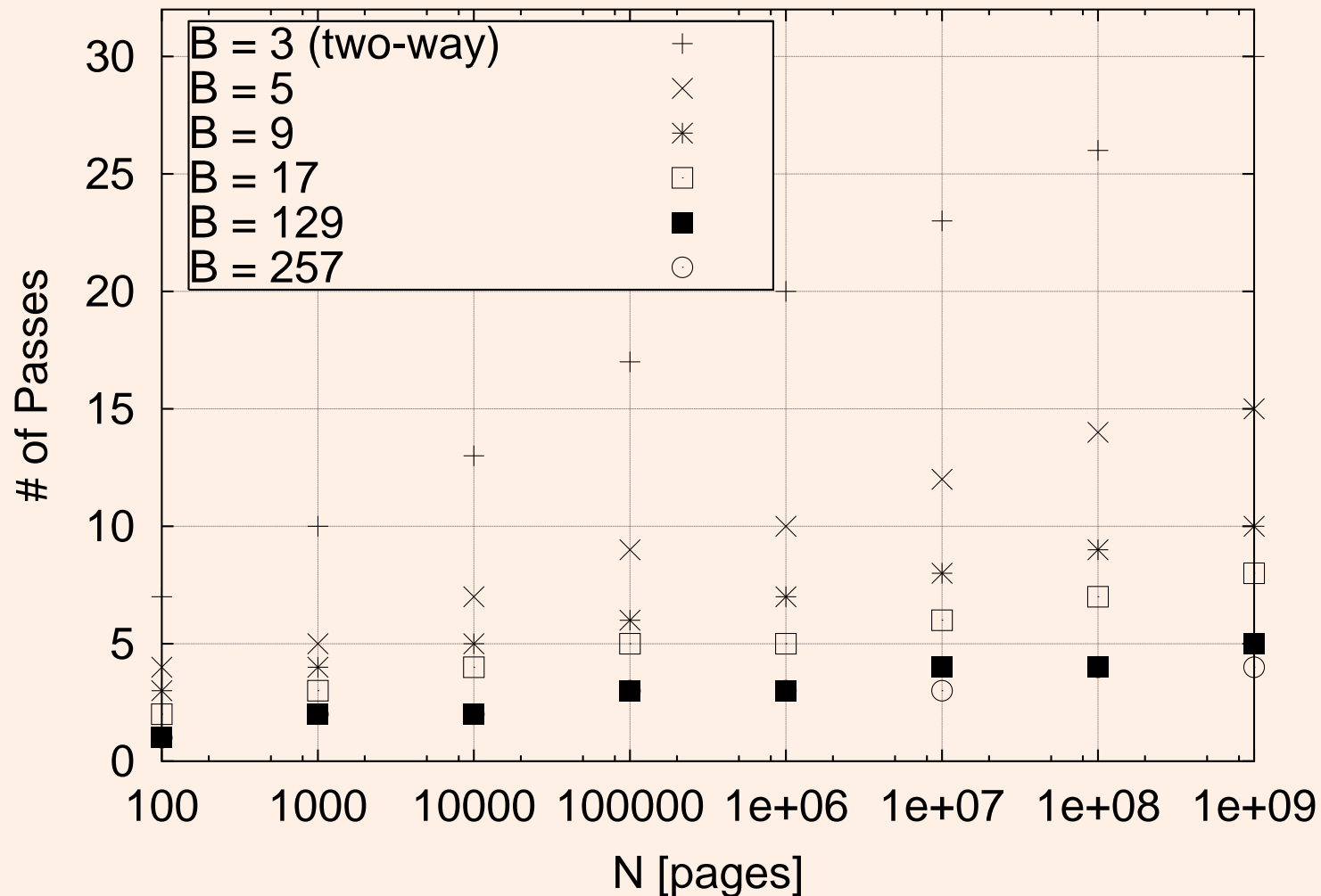
$(B - 1)$ -way merge using a buffer of B pages



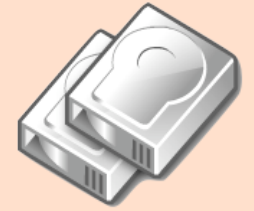
External Sorting: I/O Behavior

- The I/O savings in comparison to two-way merge sort ($B = 3$) can be substantial:

of passes for buffers of size $B = 3, 5, \dots, 257$



External Sorting: I/O Behavior



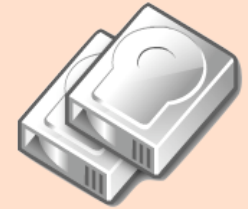
- Sorting N pages with B buffer frames requires

$$2 \cdot N \cdot \left(1 + \lceil \log_{B-1} \lceil N/B \rceil \rceil\right)$$

I/O operations.

 What is the access pattern of these I/Os?

External Sorting: I/O Behavior



- Sorting N pages with B buffer frames requires

$$2 \cdot N \cdot (1 + \lceil \log_{B-1} \lceil N/B \rceil \rceil)$$

I/O operations.

What is the access pattern of these I/Os?

- In Pass 0, we read chunks of size B **sequentially**.
- Everything else is **random access** due to the $B - 1$ way merge.
(Which of the $B - 1$ runs will contribute the next record...?)

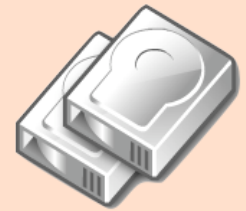
Blocked I/O

We could improve the I/O pattern by reading **blocks** of, say, b pages at once during the **merge** phases.

- Allocate b pages for each input (instead of just one).
- **Reduces per-page I/O cost** by a factor of $\approx b$.
- The price we pay is a **decreased fan-in** (resulting in an increased number of passes and more I/O operations).
- In practice, main memory sizes are typically large enough to sort files with **just one merge pass**, even with blocked I/O.

 **How long does it take to sort 8 GB (counting only I/O cost)?**

Assume 1,000 buffer pages of 8 KB each, 8.5 ms average seek time.



Blocked I/O

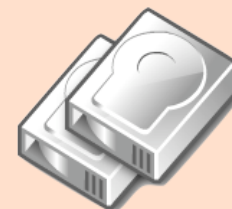
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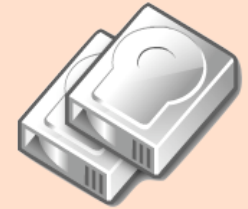
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- Without blocked I/O: $\approx 4 \cdot 10^6$ disk seeks (10.5 h) + transfer of $\approx 6 \cdot 10^6$ disk pages (13.6 min)
- With blocked I/O ($b = 32$ page blocks): $\approx 6 \cdot 32,800$ disk seeks (28.1 min) + transfer of $\approx 8 \cdot 10^6$ disk pages (18.1 min)



External Merge Sort: CPU Load and Comparisons

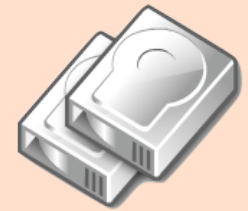


- External merge sort reduces the I/O load, but is considerably **CPU intensive**.
- Consider the $(B - 1)$ -**way merge** during passes $1, 2, \dots$:
To pick the next record to be moved to the output buffer, we need to perform $B - 2$ comparisons.

Example (Comparisons for $B - 1 = 4, \theta = <$)

{ 087 503 504 ...
170 908 994 ...
154 426 653 ...
612 613 700 ...

External Merge Sort: CPU Load and Comparisons

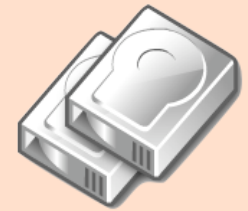


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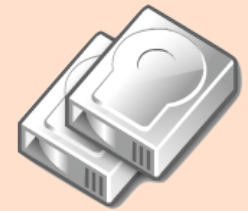


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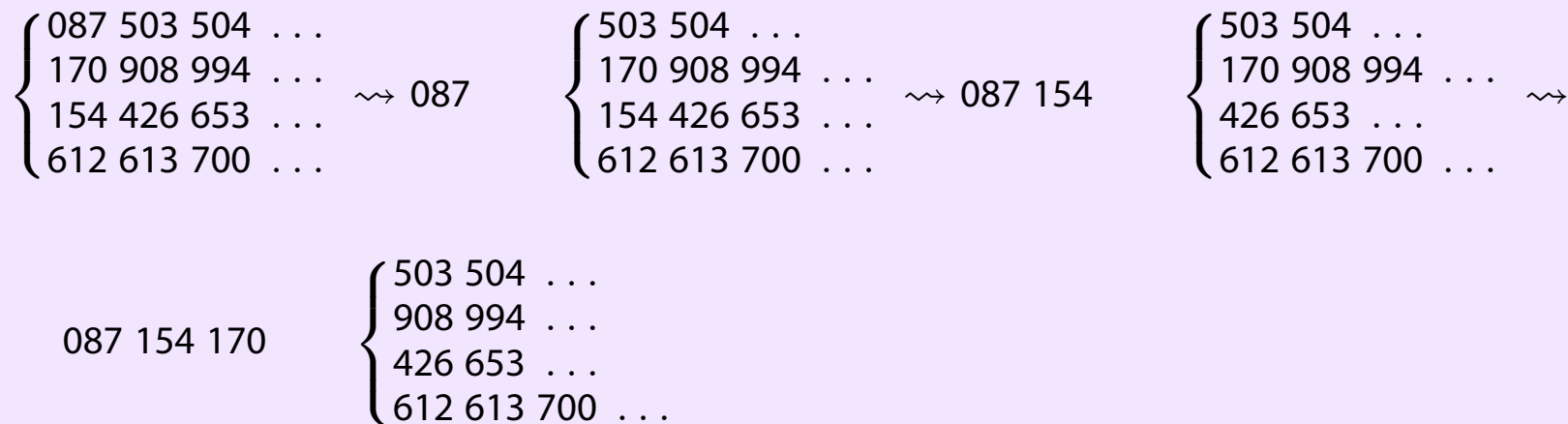
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External Merge Sort: CPU Load and Comparisons

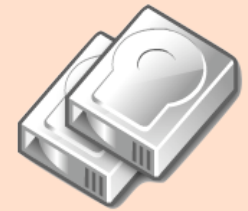


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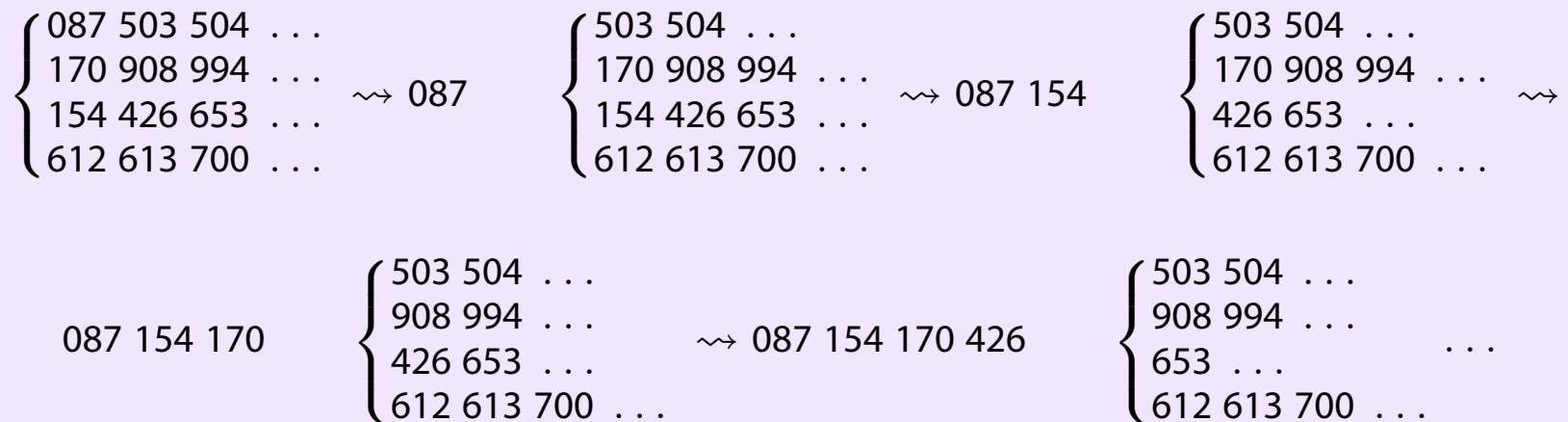


External Merge Sort: CPU Load and Comparisons

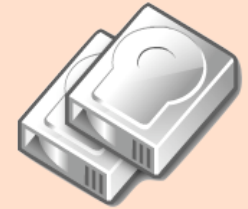


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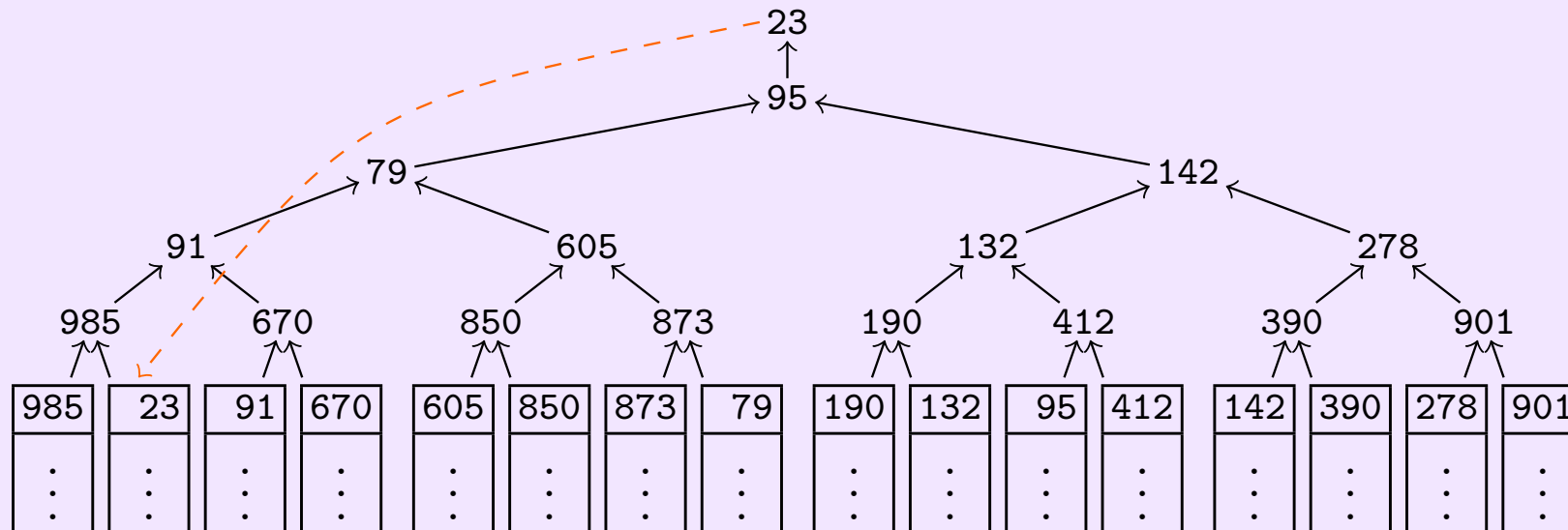
Selection Trees



Choosing the next record from $B - 1$ (or $B/b - 1$) input runs can be quite CPU intensive ($B - 2$ comparisons).

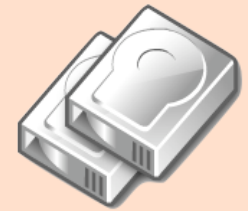
- Use a **selection tree** to reduce this cost.
- E.g., “tree of losers” (↗ D. Knuth, TAOCP, vol. 3):

Example (Selection tree, read bottom-up)



- This cuts the number of comparisons down to $\log_2 (B - 1)$.

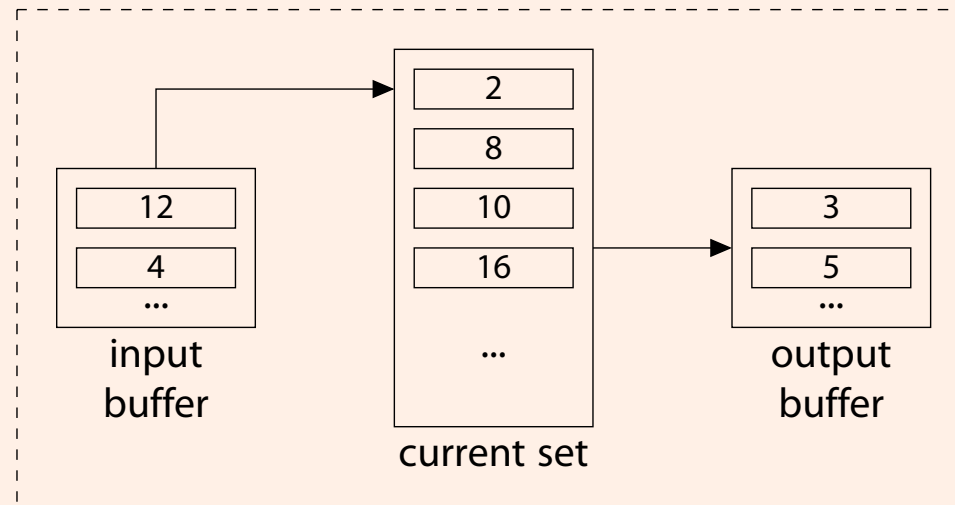
Further Reducing the Number of Initial Runs

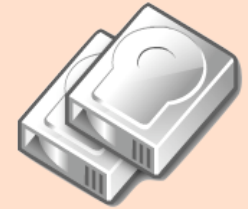


- **Replacement sort** can help to further cut down the number of initial runs $\lceil N/B \rceil$: try to **produce initial runs with more than B pages**.

Replacement sort

- Assume a buffer of B pages. Two pages are dedicated **input** and **output buffers**. The remaining $B - 2$ pages are called the **current set**:



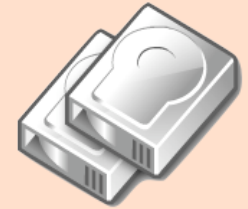


Replacement sort

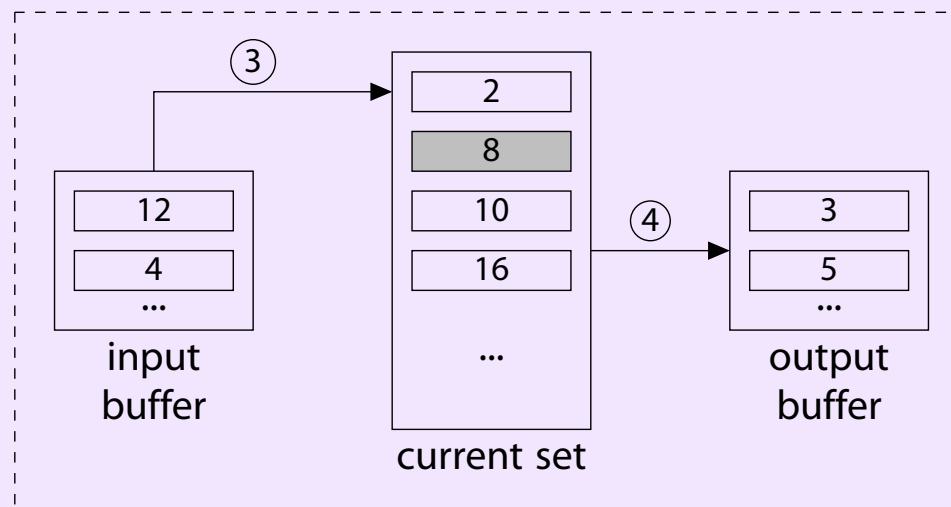
- 1 Open an empty run file for writing.
- 2 Load next page of file to be sorted into input buffer. If input file is exhausted, go to 4.
- 3 While there is space in the current set, move a record from input buffer to current set (if the input buffer is empty, reload it at 2).
- 4 In current set, pick record r with smallest key value k such that $k \geq k_{out}$ where k_{out} is the maximum key value in output buffer.¹ Move r to output buffer. If output buffer is full, append it to current run.
- 5 If all k in current set are $< k_{out}$, append output buffer to current run, close current run. Open new empty run file for writing.
- 6 If input file is exhausted, stop. Otherwise go to 3.

¹If output buffer is empty, define $k_{out} = -\infty$.

Replacement Sort

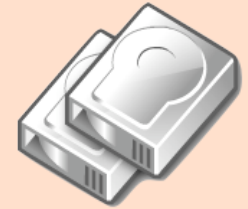


Example (Record with key $k = 8$ will be the next to be moved into the output buffer; current $k_{out} = 5$)



- The record with key $k = 2$ remains in the current set and will be written to the subsequent run.

Replacement Sort



Tracing replacement sort

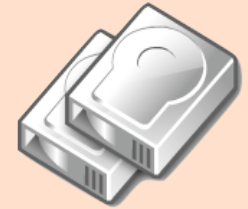
Assume $B = 6$, *i.e.*, a current set size of 4. The input file contains records with INTEGER key values:

503 087 512 061 908 170 897 275 426 154 509 612 .

Write a trace of replacement sort by filling out the table below, mark the end of the current run by $\langle \text{EOR} \rangle$ (the current set has already been populated at step **3**):

current set				output
503	087	512	061	

Replacement Sort

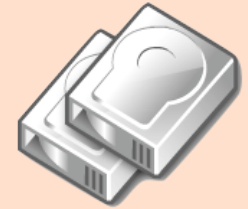


- Step 4 of replacement sort will benefit from techniques like selection tree, esp. if $B - 2$ is large.
- The replacement sort trace suggests that the length of the initial runs indeed increases. In the example: first run length $7 \approx$ **twice the size of the current set**.

Length of initial runs?

Implement replacement sort to empirically determine initial run length or check the proper analysis ([↗ D. Knuth, TAOCP, vol. 3, p. 254](#)).

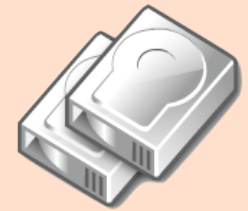
External Sort: Remarks



- External sorting follows a **divide and conquer** principle.
 - This results in a number of **indepdendent (sub-)tasks**.
 - **Execute tasks in parallel** in a distributed DBMS or exploit multi-core parallelism on modern CPUs.
- To keep the CPU busy while the input buffer is reloaded (or the output buffer appended to the current run), use **double buffering**:

Create **shadow buffers** for the input and output buffers. Let the CPU switch to the “double” input buffer as soon as the input buffer is empty and **asynchronously initiate an I/O operation** to reload the original input buffer. Treat the output buffer similarly.

(Not) Using B⁺-trees for Sorting



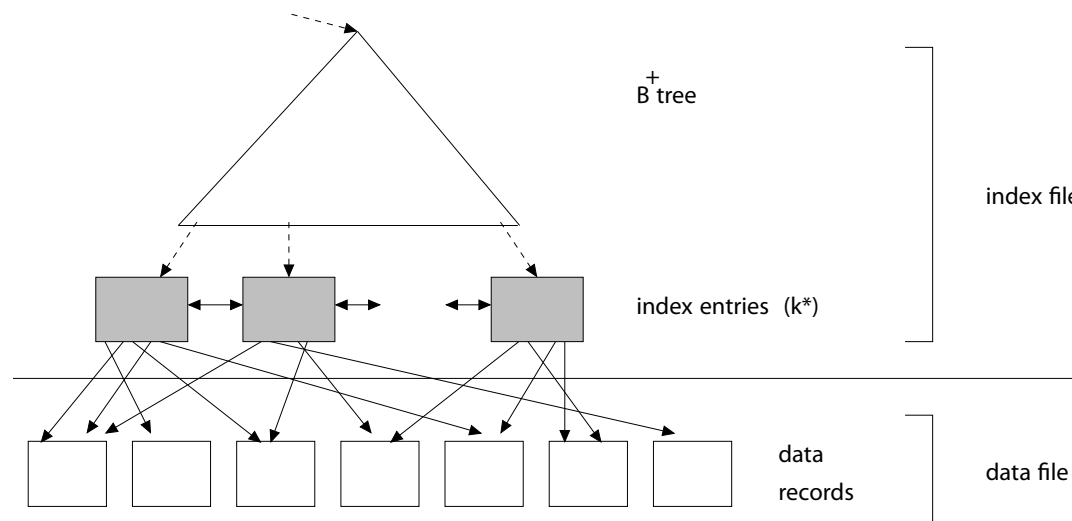
- If a B⁺-tree matches a sorting task (*i.e.*, B⁺-tree organized over key k with ordering θ), we *may* be better off to **access the index and abandon external sorting**.

① If the B⁺-tree is **clustered**, then

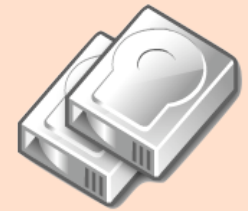
- the data file itself is already θ -sorted,
- ⇒ simply sequentially read the sequence set (or the pages of the data file).

② If the B⁺-tree is **unclustered**, then

- in the worst case, we have to initiate one I/O operation per record (not per page)!
- ⇒ do not consider the index.



(Not) Using B⁺-tree for Sorting



- Two-Way Merge Sort
- External Merge Sort
- Comparisons
- Replacement Sort
- B⁺-trees for Sorting**

- Let p denote the number of records per page (typically, $p = 10, \dots, 1000$). Expected of I/O operations to sort via an *unclustered* B⁺-tree will thus be $p \cdot N$:

Expected sort I/O operations (assume $B = 257$)

