Embedding a Functional Hybrid Modelling Language in Haskell

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- Functional Hybrid Modelling (**FHM**) is a new approach to the design of **modelling and simulation** (M&S) languages
- **Haskell** is a purely **functional programming language**

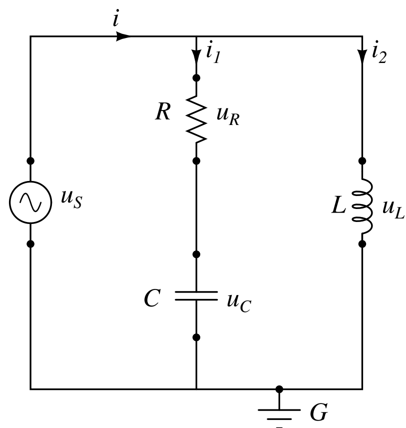
Outline

Modelling and Simulation Languages

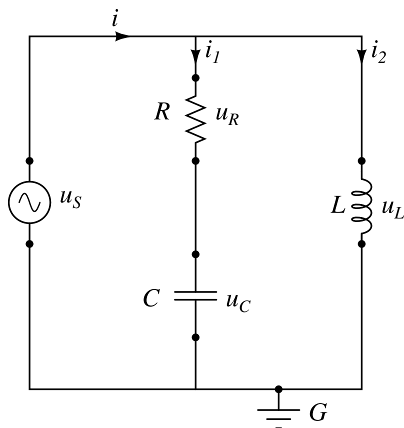
FHM as a Domain Specific Language Embedded in Haskell

Future Work and Conclusions

Simple Electrical Circuit



Simple Electrical Circuit



Differential Algebraic Equations
(**DAE**s)

$$f(\frac{d\vec{x}}{dt}, \vec{x}, \vec{y}, t) = 0$$

$$\begin{aligned}
u_S &= sin(2\pi t) \\
u_R &= R \cdot i_1 \\
i_1 &= C \cdot \frac{du_C}{dt} \\
u_L &= L \cdot \frac{di_2}{dt} \\
i_1 + i_2 &= i \\
u_R + u_C &= u_s \\
u_S &= u_L
\end{aligned}$$

M&S with General Purpose Programming Languages

Ordinary Differential Equation (**ODE**) in **explicit** form

$$\frac{d\vec{x}}{dt} = f(\vec{x}, t)$$

$$
\begin{aligned}
u_S &= sin(2\pi t) \\
u_L &= u_S \\
\frac{di_2}{dt} &= \frac{u_L}{L} \\
u_R &= u_S - u_C \\
i_1 &= \frac{u_R}{R} \\
\frac{du_C}{dt} &= \frac{i1}{C} \\
i &= i_1 + i_2
\end{aligned}
$$

M&S with General Purpose Programming Languages

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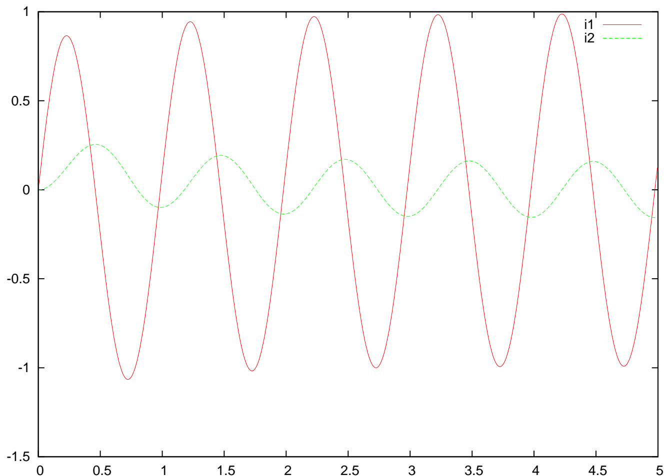
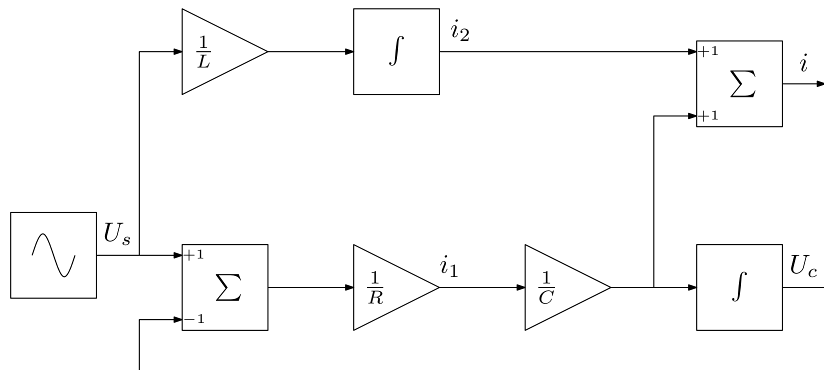
\end{aligned}$$

```
integrateSimpleCircuit dt r c l = go 0 0 0
  where go t uc i2 = hd : tl
          where us  = sin (2 * pi * t)
                ul  = us
                di2 = (ul / l) * dt
                ur  = us - uc
                i1  = ur / r
                duc = (i1 / c) * dt
                i   = i1 + i2
                hd  = (t, i, i1, i2, us, ur, uc, ul)
                tl  = go (t + dt) (uc + duc) (i2 + di2)
```

Simulation Result



Causal Modelling Languages (e.g. Simulink)



Causal Modelling Languages

- Very complex/complicated
- Not Modular
- Not Reusable

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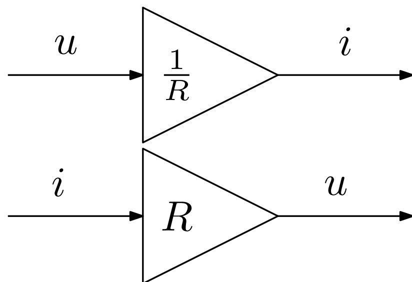
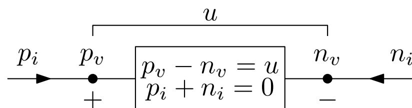
$$i = \frac{u}{R}$$

$$u = R \cdot i$$



Non-causal modelling using FHM Approach



$$
\begin{aligned}
twoPin &= \quad \textbf{sigrel}\ ((\textbf{flow}\ p_i, p_v), (\textbf{flow}\ n_i, n_v), u)\ \textbf{where} \\
&\qquad p_v - n_v = u \\
&\qquad p_i + n_i = 0 \\
resistor &= \lambda r \rightarrow \textbf{sigrel}\ ((\textbf{flow}\ p_i, p_v), (\textbf{flow}\ n_i, n_v))\ \textbf{where} \\
&\qquad twoPin \diamond ((p_i, p_v), (n_i, n_v), u) \\
&\qquad r * p_i = u \\
resistor &= \lambda r \rightarrow \textbf{sigrel}\ ((\textbf{flow}\ p_i, p_v), (\textbf{flow}\ n_i, n_v))\ \textbf{where} \\
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\mathit{resistor} &= \lambda r \rightarrow \textbf{sigrel}\ ((\textbf{flow}\ p_i, p_v), (\textbf{flow}\ n_i, n_v))\ \textbf{where} \\
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&\qquad r * p_i = u \\
\mathit{inductor} &= \lambda l \rightarrow \textbf{sigrel}\ ((\textbf{flow}\ p_i, p_v), (\textbf{flow}\ n_i, n_v))\ \textbf{where} \\
&\qquad \mathit{twoPin} \diamond ((p_i, p_v), (n_i, n_v), u) \\
&\qquad l * \mathit{der}\ p_i = u \\
\mathit{capacitor} &= \lambda c \rightarrow \textbf{sigrel}\ ((\textbf{flow}\ p_i, p_v), (\textbf{flow}\ n_i, n_v))\ \textbf{where} \\
&\qquad \mathit{twoPin} \diamond ((p_i, p_v), (n_i, n_v), u) \\
&\qquad c * \mathit{der}\ u = p_i
\end{aligned}
$$

Non-causal modelling using FHM Approach

$$
\begin{aligned}
simpleCircuit = \textbf{sigrel}\ (\textbf{flow}\ i, u)\ \textbf{where}\quad & \\
& vSourceAC\ 1\ 1 \diamond ((acp_i, acp_v), (acn_i, acn_v)) \\
& resistor\ 1 \diamond ((rp_i, rp_v), (rn_i, rn_v)) \\
& inductor\ 1 \diamond ((lp_i, lp_v), (ln_i, ln_v)) \\
& capacitor\ 1 \diamond ((cp_i, cp_v), (cn_i, cn_v)) \\
& ground \diamond (gp_i, gp_v) \\
& \textbf{connect}\ acp_i\ rp_i\ lp_i \\
& \textbf{connect}\ acp_v\ rp_v\ lp_v \\
& \textbf{connect}\ rn_i\ cp_i \\
& \textbf{connect}\ rn_v\ cp_v \\
& \textbf{connect}\ acn_i\ cn_i\ ln_i\ gp_i \\
& \textbf{connect}\ acn_v\ cn_v\ ln_v\ gp_v \\
& i = acp_i \\
& u = acp_v - acn_v
\end{aligned}
$$

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& capacitor\ 1 \diamond ((cp_i, cp_v), (cn_i, cn_v)) \\
& ground \diamond (gp_i, gp_v) \\
& acp_i + rp_i + lp_i = 0 \\
& acp_v = rp_v = lp_v \\
& rn_i + cp_i = 0 \\
& rn_v = cp_v \\
& acn_i + cn_i + ln_i + gp_i = 0 \\
& acn_v = cn_v = ln_v = gp_v \\
& i = acp_i \\
& u = acp_v - acn_v
\end{aligned}
$$

Functional Hybrid Modelling (FHM)

- FHM extends a purely functional language with a few key abstractions (e.g. signal relation) for supporting non-causal modelling [Nilsson et al. PADL2003]
- FHM approach is more **declarative** and **high-level**, i.e. focuses on **what** to model rather then **how** to simulate
  - Modular
  - Reusable

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- FHM extends a purely functional language with a few key abstractions (e.g. signal relation) for supporting non-causal modelling [Nilsson et al. PADL2003]
- FHM approach is more **declarative** and **high-level**, i.e. focuses on **what** to model rather then **how** to simulate
  - Modular
  - Reusable
- Unfortunately, there was no publicly available implementation available.
- We investigate the implementation of FHM as a Haskell embedded domain-specific language ((EDSL)) in Haskell.

Domain Specific Embedding using Abstract Syntax Trees (ASTs)

$$
\begin{aligned}
&\textbf{data}\ \mathit{SigRel} = \mathit{SigRel}\ \mathit{Pattern}\ [\mathit{Equation}] \\
&\textbf{data}\ \mathit{Pattern} = \ldots \\
&\textbf{data}\ \mathit{Equation} = \\
&\quad\ \mathit{EquationEq}\ \mathit{Expr}\ \mathit{Expr} \\
&\quad | \mathit{EquationSigRelApp}\ \mathit{SigRel}\ \mathit{Expr} \\
&\textbf{data}\ \mathit{Expr} = \ldots
\end{aligned}
$$

Originally Proposed Syntax

$$
\begin{aligned}
&\mathit{twoPin} = \\
&\quad \textbf{sigrel}\ ((\textbf{flow}\ p_i, p_v), (\textbf{flow}\ n_i, n_v), u)\ \textbf{where} \\
&\qquad p_v - n_v = u \\
&\qquad p_i + n_i\ \ = 0 \\
&\mathit{resistor}\ r = \\
&\quad \textbf{sigrel}\ ((\textbf{flow}\ p_i, p_v), (\textbf{flow}\ n_i, n_v))\ \textbf{where} \\
&\qquad \mathit{twoPin} \diamond ((p_i, p_v), (n_i, n_v), u) \\
&\qquad r * p_i = u
\end{aligned}
$$

Executable Haskell Code that Uses Quasiquoting

$$
\begin{array}{l}
\textbf{type}\ Pin = (\mathbb{R}, \mathbb{R}) \\
twoPin :: SR\ (Pin, Pin, \mathbb{R}) \\
twoPin = [\textbf{\$fhm}| \\
\quad \textbf{sigrel}\ ((\textbf{flow}\ p_i, p_v), (\textbf{flow}\ n_i, n_v), u)\ \textbf{where} \\
\qquad p_v - n_v = u \\
\qquad p_i + n_i \ \ = 0 \\
|] \\
resistor :: \mathbb{R} \rightarrow SR\ (Pin, Pin) \\
resistor\ r = [\textbf{\$fhm}| \\
\quad \textbf{sigrel}\ ((\textbf{flow}\ p_i, p_v), (\textbf{flow}\ n_i, n_v))\ \textbf{where} \\
\qquad \$twoPin\$ \diamond ((p_i, p_v), (n_i, n_v), u) \\
\qquad \$r\$ * p_i = u \\
|]
\end{array}
$$

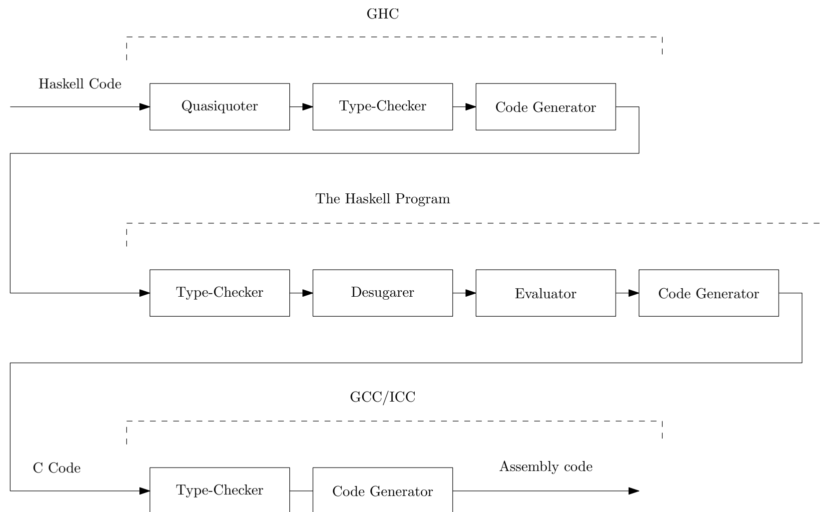
Complete Model in Haskell

$$
\begin{array}{l}
\mathit{simpleCircuit} :: \mathit{SR}\ (\mathbb{R}, \mathbb{R}) \\
\mathit{simpleCircuit} = [\$\mathbf{fhm}| \\
\quad \mathbf{sigrel}\ (\mathbf{flow}\ i, u)\ \mathbf{where} \\
\qquad \$\mathit{vSourceAC}\ 1\ 1\$ \diamond ((\mathit{acp}_i, \mathit{acp}_v), (\mathit{acn}_i, \mathit{acn}_v)) \\
\qquad \$\mathit{resistor}\ 1\$ \diamond ((\mathit{rp}_i, \mathit{rp}_v), (\mathit{rn}_i, \mathit{rn}_v)) \\
\qquad \$\mathit{inductor}\ 1\$ \diamond ((\mathit{lp}_i, \mathit{lp}_v), (\mathit{ln}_i, \mathit{ln}_v)) \\
\qquad \$\mathit{capacitor}\ 1\$ \diamond ((\mathit{cp}_i, \mathit{cp}_v), (\mathit{cn}_i, \mathit{cn}_v)) \\
\qquad \$\mathit{ground}\$ \diamond (\mathit{gp}_i, \mathit{gp}_v) \\
\qquad \mathbf{connect}\ \mathit{acp}_i\ \mathit{rp}_i\ \mathit{lp}_i \\
\qquad \mathbf{connect}\ \mathit{acp}_v\ \mathit{rp}_v\ \mathit{lp}_v \\
\qquad \mathbf{connect}\ \mathit{rn}_i\ \mathit{cp}_i \\
\qquad \mathbf{connect}\ \mathit{rn}_v\ \mathit{cp}_v \\
\qquad \mathbf{connect}\ \mathit{acn}_i\ \mathit{cn}_i\ \mathit{ln}_i\ \mathit{gp}_i \\
\qquad \mathbf{connect}\ \mathit{acn}_v\ \mathit{cn}_v\ \mathit{ln}_v\ \mathit{gp}_v \\
\qquad i = \mathit{acp}_i \\
\qquad u = \mathit{acp}_v - \mathit{acn}_v \\
|]
\end{array}
$$

Meta-modelling and Higher-order Models

$$
\begin{array}{l}
\mathit{serialise} :: [\mathit{SR}\ (\mathit{Pin}, \mathit{Pin})] \rightarrow \mathit{SR}\ (\mathit{Pin}, \mathit{Pin}) \\
\mathit{serialise}\ [\mathit{sr}] = \mathit{sr} \\
\mathit{serialise}\ (\mathit{sr} : \mathit{srs}) = [\mathbf{\$fhm}| \\
\quad \mathbf{sigrel}\ ((\mathbf{flow}\ p_i, p_v), (\mathbf{flow}\ n_i, n_v))\ \mathbf{where} \\
\qquad \$\mathit{sr}\$ \qquad\qquad\quad\ \diamond ((p_i, p_v), (n1_i, n1_v)) \\
\qquad \$\mathit{serialise}\ \mathit{srs}\$ \diamond ((p2_i, p2_v), (n_i, n_v)) \\
\qquad \mathbf{connect}\ n1_i\ p2_i \\
\qquad \mathbf{connect}\ n1_v\ p2_v \\
|]
\end{array}
$$

Code Flow in the Implementation



Summary of Implementation

- The method of embedding was inspired by [Mainland, ICFP 2008]
- It employs new features introduced in the Glasgow Haskell Compiler (GHC) version 6.10
- In particular, our embedding uses quasiquoting that allowed us to embed the language using the domain specific syntax
- Generates fast simulation code in C

Future Work and Conclusions

-
  - **F**unctional +
  - **H**ybrid -
  - **M**odelling +

Future Work and Conclusions

- - **F**unctional +
  - **H**ybrid -
  - **M**odelling +
- The work is supported by the first publicly available prototype implementation of a FHM language.
- The source code (under the open source BSD license) and paper is available online (`http://www.cs.nott.ac.uk/~ggg/`).

Thank You!